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*Jorge M. López*, Departamento de Matemáticas, Universidad de Puerto Rico en Río Piedras.

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*Rémi Mégret*, Department of Computer Science, University of Puerto Rico at Río Piedras.

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# 1 Itinerario Global (Global Schedule)

Viernes (Friday)						
Hora (Time)	Student Center	CA 205	CA 206	CA 210	CA 211	CA 204
12:00-2:00	Registration					
1:00-5:30	Exhibit Booths					
1:00-1:25			S. Mattson			
1:30-1:55		J. Gómez	A. Gómez	I. Rubio	E. Orozco	
2:00-2:25		A. Masuda	F. Piñero	O. González	C. Caiseda	
2:30-2:55		L. Cáceres		L. Sepulveda	L. Hernández	
3:00-3:25	Coffee Break					
3:30-3:55		E. Orozco	M. Delgado	D. Gómez	A. Portnoy	
4:00-4:25		C. Serna	H. Janwa	R. Arce	J. Gómez	
4:30-5:20		L. Medina (CA Amphitheater)				
5:30-6:00		Buses to Lito Peña Art Center				
6:00-9:00		SIDIM 2019 Official Opening (cocktail and dinner)				
9:00-9:30		Buses back to UPRH				
Sábado (Saturday)						
Hora (Time)	UPRH Theater	CA 205	CA 206	CA 210	CA 211	CA 204
7:30-10:00	Registration					
8:00-4:00	Exhibit Booths					
7:30-9:00		Desayuno (breakfast) - Student Center				
8:00-8:25		R. Aparicio	P. Negrón	O. Cruz	S. Pérez	
8:30-8:55		C. Carvajal	S. Nguyen	E. Ramos	M. Robles	
9:00-9:25		G. Ferrer	J. Rivera	J. Chan	J. La Luz	P. Cholak
9:30-9:55		V. Díaz	M. Rosado	H. Carbia	G. Salvia	F. Piñero
10:00-11:00	Afiches (Posters)					
11:00-11:50	C. Moreno					
12:00-1:00		Almuerzo (Lunch) - UPRH Cafeteria				
1:00-1:50	D. Panario					
2:00-2:25		J. González	A. Bustillo	I. Rodríguez	R. Ortiz	A. Lecompte
2:30-2:55		J. Henríquez	M. Cruz	O. Medina	A. Hernández	L. Pericchi
3:00-3:25		J. Pastrana	M. Marciano	J. Sotero	C. López	M.R. Emamy
3:30-3:55		A. Vélez	J. Vega		J. Calderón	
4:00-4:30		Sesión Administrativa (Administrative Session) (CA Amphitheater)				

## 2 Itinerario Detallado (Detailed Schedule)

Horario	Lugar	Actividad
<b>Viernes</b>		
12:00-2:00	Student Center	<b>Registration</b>
1:00-5:30	Student Center	<b>Exhibit Booths</b>
1:00-1:25		<b>Contributed Presentations</b>
	CA 210	<i>Working with Francis Castro</i> S. Mattson
1:30-1:55		<b>Contributed Presentations</b>
	CA 205	<i>On cyclotomic subfields</i> J. Gómez
	CA 206	<i>Linear codes, graphs, and the projective special linear group</i> A. Gómez
	CA 210	<i>Extension of the covering method to any finite field</i> I. Rubio
	CA 211	<i>Explorando la Ciencia de Cómputos para Puerto Rico: algunas experiencias y resultados del proyecto ECS4PR*</i> E. Orozco
2:00-2:25		<b>Contributed Presentations</b>
	CA 205	<i>Permutation polynomials over finite fields</i> A. Masuda
	CA 206	<i>Binary quantum codes from the Hermitian curve</i> F. Piñero
	CA 210	<i>An improvement to Chevalley's theorem with restricted variables</i> O. Gonzalez
	CA 211	<i>MSEIP-INTERactua: a student success network for STEM students in first-year mathematics courses</i> C. Caiseda
2:30-2:55		<b>Contributed Presentations</b>
	CA 205	<i>Problema de pertenencia en ideales de un dominio de polinomios</i> L. Cáceres
	CA 206	<i>Some new result on the covering radius of nested MDS codes</i> J. Orozco
	CA 210	<i>Recursions associated to trapezoid, symmetric and rotation symmetric functions over Galois fields</i> L. Sepulveda

Horario	Lugar	Actividad
	CA 211	<i>Cuantificación de la ventaja cognitiva de estudiar límites de funciones utilizando infinitésimos y Cálculo no Estándar con estudiantes del curso Métodos Cuantitativos II para Administración de Empresas</i> L. Hernández
3:00-3:25	Student Center	<b>Coffee Break</b>
3:30-3:55		<b>Contributed Presentations</b>
	CA 205	<i>Fast FPGA implementations of elliptic curve point multiplication for a family of special finite fields</i> E. Orozco
	CA 206	<i>At most <math>N</math>-to-one functions over finite fields for new trap-door functions</i> M. Delgado
	CA 210	<i>Linear complexity analysis of multidimensional periodic arrays</i> D. Gómez
	CA 211	<i>Music: Art, Physics and Mathematics, a practical example of interdisciplinary teaching and learning</i> A. Portnoy
4:00-4:25		<b>Contributed Presentations</b>
	CA 205	<i>Value distribution of elementary symmetric polynomials and their perturbations over finite fields</i> C. Serna
	CA 206	<i>Public key cryptosystems based on AG codes suitable for post- quantum PKC</i> H. Janwa
	CA 210	<i>Some computational contributions to help accelerate Dr. Castro's research</i> R. Arce
	CA 211	<i>Classic functions, an analytic approach</i> J. Gómez
4:30-5:20	CA Amphitheatre	<b>Invited Presentation:</b> <i>Boolean functions, recursions and coffee: a journey into the world of exponential sums</i> Luis Medina
5:30-6:00	UPRH Roundabout	<b>Buses to Lito Peña Art Center</b>
5:30-6:30	Lito Peña Art Center	<b>Cocktail</b>
6:30-8:00	Lito Peña Art Center	<b>SIDIM 2019 Opening Ceremony</b>
8:00-9:00	Lito Peña Art Center	<b>Dinner</b>
9:00-9:30	Lito Peña Art Center	<b>Buses back to UPRH</b>

Horario	Lugar	Actividad
<b>Sábado</b>		
7:30-10:00	UPRH Theater	<b>Registration</b>
8:00-4:00	UPRH Theater	<b>Exhibit Booths</b>
7:30-9:00	Student Center	<b>Breakfast</b>
8:00-8:25		<b>Conferencias Concurrentes</b>
	CA 205	<i>Besov maximal regularity for a class of degenerate integro-differential equations with infinite delay in Banach spaces</i> R. Aparicio
	CA 206	<i>Infinite energy cavitating solutions: stabilization of the energy functional and its numerical approximation</i> P. Negrón
	CA 210	<i>Clustering methods to classify a broad range of species from their vocalization</i> O. Cruz
	CA 211	<i>Existence and Properties of Maximal Metrics</i> S. Pérez
8:30-8:55		<b>Conferencias Concurrentes</b>
	CA 205	<i>The anisotropic parabolic problem with Wentzell boundary conditions and variable exponents</i> C. Carvajal
	CA 206	<i>Stochastic maximum principle for controlled switching diffusions with conditional mean-field</i> S. Nguyen
	CA 210	<i>Clasificación de Microfotografías de Epidermis de Hojas utilizando Redes Neuronales artificiales</i> E. Ramos
	CA 211	<i>On an infinite family of satellite knots and its knot polynomials</i> M. Robles
9:00-9:25		<b>Conferencias Concurrentes</b>
	CA 205	<i>The Koch cube domain, a 3-dimensional Koch snowflake analogue</i> G. Ferrer
	CA 206	<i>A geometric proof for the existence of travelling wave solutions for a model of epidemic wounds</i> J. Rivera
	CA 210	<i>Learning good features to discriminate untagged bees in video using non-supervised learning</i> J. Chan
	CA 211	<i>Criptografía simplicial</i> J. La Luz
	CA 203	<i>Computability-theoretic aspects of Ramsey's Theorem</i> P. Cholak

Horario	Lugar	Actividad
9:30-9:55		<b>Conferencias Concurrentes</b>
	CA 205	<i>The elliptic anisotropic problem with Wentzell boundary conditions and variable exponents</i> V. Díaz
	CA 206	<i>Modeling ultrafine particulate matter through motor vehicles emissions</i> M. Rosado
	CA 210	<i>Convolutional deep learning to predict physical characteristics from genomic data</i> H. Carbia
	CA 211	<i>Implementación de criptografía simplicial</i> G. Salivia
	CA 203	<i>Optimal locally recoverable codes of distance 5 from affine cartesian codes</i> F. Piñero
10:00-11:00	UPRH Theater	<b>Posters and Coffee Break</b>
		<i>Matrices with good cross-correlation and autocorrelation for watermarking 2d digital images</i> Cesar F. Bolaños Revelo
		<i>DevSecOps: continuous integration meets containers security</i> Jordan A. Caraballo-Vega
		<i>Mathematical Modeling with Human Mobility of the ZIKA virus in Puerto Rico</i> Nevil Clavell
		<i>Analysis of social factors influencing heroin addiction</i> Giovanni G. Colón-Cabezudo
		<i>A Mathematical Modeling Analysis for Evaluating Cost-Effective Treatment Strategies for the Control of HSV-2 Infection in the U.S.</i> Mónica Colón-Vargas
		<i>Binomials of the form <math>x^m[x^{\frac{q-3}{2}} + A]</math> that do not produce permutations</i> Dylan Cruz
		<i>Phenotype response to higher order mutation interactions, a spectral analysis approach</i> Lillian González Albino
		<i>Recognition of Fanning Bees from Video using Convolutional Neural Networks</i> Edward Latorre
		<i>Deriving near-surfaces bulk densities of near-earth asteroids using radar observations</i> Andy J. López-Oquendo

Horario	Lugar	Actividad
		<i>Un modelo matemático para evaluar los efectos de la herbivoría de la polilla (Oidaematophorus Espeletiae) sobre el frailejón (Espeletiae Grandiflora)</i> Jessenia Ivonne Quintero Guiza
		<i>On the number of solutions to systems of polynomial equations with multivariate polynomials over finite fields as coefficients</i> Carlos E. Seda Damiani
		<i>Introduction to f-factorization</i> Geraldo E. Soto-Rosa
		<i>Some reduced linear modular systems and their structures</i> María del C. Ramos Álamos
		<i>Análisis de datos de simuladores de conducción aplicando modelos de regresión funcional</i> Daniel M. Rocha Clavijo
		<i>Predicción de lluvia en Puerto Rico: un enfoque con redes neuronales artificiales</i> Efraín Vargas Ramos
		<i>Prior-free Bayes factors based on data splitting compared with geometric intrinsic Bayes factors</i> Yekun Wang
11:00-11:50	UPRH Theater	<b>Invited Presentation:</b> <i>Zeta and L-functions over finite fields: en memoria de Francis Castro</i> Carlos Moreno
12:00-1:00	UPRH Cafeteria	<b>Lunch</b>
1:00-1:50	UPRH Theater	<b>Invited Presentation:</b> <i>The research contributions of Francis Castro</i> Daniel Panario
2:00-2:25		<b>Conferencias Concurrentes</b>
	CA 205	<i>Fundamental solutions for discrete dynamical systems involving the fractional Laplacian</i> J. Gonzalez
	CA 206	<i>New families of 3D watermaks</i> A. Bustillo
	CA 210	<i>Automatic monitoring of the foraging behavior of tagged and untagged honey bees</i> I. Rodriguez
	CA 211	<i>La teoría de las <math>\tau</math>-factorizaciones</i> R. Ortiz
	CA 203	<i>Comparison of the frequencies of numbers free of small primes, and squares or cubes, with their probabilities inferred from Eulers identity for the harmonic series</i> A. Lecompte

<b>Horario</b>	<b>Lugar</b>	<b>Actividad</b>
2:30-2:55		<b>Conferencias Concurrentes</b>
	CA 205	<i>A problem of Ambrosetti-Prodi type for the anisotropic Laplace operator</i> J. Henríquez
	CA 206	<i>Analytical investigation of basic reproductive ratio in designing disease screen-treat programs</i> M. Cruz
	CA 210	<i>Puerto Rico crime cap, a visualization and analysis of a geographical dataset</i> O. Medina
	CA 211	<i>On the characterization of <math>\tau_{(n)}</math>-atoms</i> A. Hernández
	CA 203	<i>Redefine Statistical Significance</i> L. Pericchi
3:00-3:25		<b>Conferencias Concurrentes</b>
	CA 205	<i>Non Uniform Dependence for Euler Equations in Besov Spaces</i> J. Pastrana
	CA 206	<i>Modeling coffee fruit infestation by the coffee berry borer</i> M. Marcano
	CA 210	<i>Movimientos migratorios en Puerto Rico de 1850 a 1984</i> J. Sotero
	CA 211	<i>Los grafos de <math>\tau_{(n)}</math>-divisores <math>\tau_{(n)}</math>-irreducibles</i> C. López
	CA 203	<i>The 5-cube cut number problem; a short proof for a basic lemma</i> M.R. Emany
3:30-3:55		<b>Conferencias Concurrentes</b>
	CA 205	<i>Solvability over <math>C(\bar{\Omega})</math> of the parabolic anisotropic Robin problem with variable exponents</i> A. Velez
	CA 206	<i>Dynamics of Nonlinear Wave Propagation in Hyperbolic Metamaterials</i> J. Vega
	CA 211	<i>La aplicación de funciones a la teoría de <math>\tau</math>-factorizaciones</i> J. Calderón
4:00-4:30	CA 211	<b>Administrative Session</b>

### 3 Charlas Plenarias / Plenary Talks

#### Zeta and L-functions over finite fields: en memoria de Francis Castro

*Carlos Moreno*, City University of New York

Zeta functions over finite fields were first studied by E. Artin in his thesis. Andre Weil extended their study to algebraic curves and proved the first Riemann Hypothesis for them. In the latter part of the Twentieth Century these zeta and L-functions found many important applications to coding theory and cryptography.

The above background set the trend for the study of zeta and L-functions when Francis Castro began his mathematical career and marks the beginning of Francis's particular interests in the study of computational aspects related to the problem of counting the number of solutions to algebraic equations over finite fields.

In this talk we shall discuss some aspects of Francis work:

- I. "Last Entry in Gauss Diary"
- II. Sharp Bounds for mixed exponential Sums,
- III. Sign Changes For Kloosterman Sums
- IV. Space Complexity of Calculating Zeta Functions ( in the spirit of Chevalley-Waring)

It is our hope that the (partial ) discussion of this short list will illuminate the beauty of the subject and illustrate Francis's mathematical legacy.

#### The research contributions of Francis Castro

*Daniel Panario*, Carleton University, Canada

In this talk I will revise the research contributions of Francis Castro. I will also comment on some personal encounters with Francis.

Most of Francis Castro's works are in finite fields and number theory, but he also worked on some combinatorial problems. I will discuss in some detail Francis important contributions in areas such as exponential sums and diagonal equations over finite fields, and will also touch other areas that he has worked like Dickson polynomials.

I will also comment on my relation with Francis both in research (for example, co-organizer of a workshop, Francis work on the Handbook of Finite Fields, etc) as well as on my personal contacts with him along the years.

## Boolean functions, recursions and coffee: a journey into the world of exponential sums

*Luis A. Medina*, University of Puerto Rico at Río Piedras

Boolean functions are part of a beautiful branch of combinatorics with applications to scientific fields like cryptography, theory of error-correcting codes and information theory. In some applications related to cryptography it is important for Boolean functions to be balanced. Balancedness of Boolean functions can be studied from the point of view of exponential sums. In this talk, I discuss the journey of Dr. Castro and me in this area of research and how the discovery that exponential sums of symmetric Boolean functions are linear recurrent influenced our research.

## 4 Charlas Concurrentes / Concurrent Talks

(In alphabetical order using the last name of the speaker.)

### Besov maximal regularity for a class of degenerate integro-differential equations with infinite delay in Banach spaces

*Rafael Aparicio*, Statistical Institute and Computerized Information Systems, University of Puerto Rico at Río Piedras.

*Valentin Keyantuo*, Department of Mathematics, University of Puerto Rico at Río Piedras.

The theory of operator valued Fourier multipliers is used to obtain characterizations for well-posedness of a large class of degenerate integro-differential equations of second order in time in Banach spaces. We concentrate on the case of Besov spaces. Maximal regularity is an important tool for the treatment of nonlinear equations. The equations under consideration are important in several applied problems in physics and material science, in particular for phenomena where memory effects are important. A notable class of such equations arise in viscoelasticity.

**Acknowledgements:** The work of the second author is partially supported by the Air Force Office of Scientific Research (AFOSR) under Award No: FA9550-18-1-0242.

**Keywords:** well-posedness, maximal regularity, operator-valued Fourier multiplier, Besov spaces

### Some computational contributions to help accelerate Dr. Castro's research

*Rafael Arce-Nazario*, Department of Computer Science, University of Puerto Rico at Río Piedras.

*José Ortiz-Ubarri*, Department of Computer Science, University of Puerto Rico at Río Piedras.

In this presentation we will talk about our experiences working with Dr. Francis Castro and our computational contributions to help accelerate his research. It is well known that many ideas and

theories can be easily discarded by finding a counterexample. Furthermore, to the best of our recollection, many of Dr. Castro's theories started as an intuition that was enhanced by working on incremental examples and analyzing their patterns. Sometimes finding those counterexamples and/or generating enough data to find patterns can be done by hand, or by simple programming; but often times problems are not so easy to program due to resource and system constraints. Dr. Castro was known to be a good programmer but, to our fortune, he sometimes sought our help with problems that required more elaborate computational solutions, e.g. exploiting architectural properties of systems, using high performance computing, improving algorithmic efficiency, or a combination of them. We will discuss a few algorithms that we implemented to assist in his research topics in polynomial permutations, Egyptian fractions, and binary cyclic codes. The algorithms use computer programming techniques such as branch and bound, bitwise manipulation, and finite field operations.

**Keywords:** algorithms, high performance computing, polynomial permutations, Egyptian fractions, binary cyclic codes

## New families of 3D watermarks

*Alcibiades Bustillo*, Department of Mathematics, University of Puerto Rico at Mayagüez.  
*Dorothy Bollman*, Department of Mathematics, University of Puerto Rico at Mayagüez.

We construct a new families of 3D watermarks by composing the 2-dimensional Legendre array with shift sequences derived from cyclic groups of points on a elliptic curve over  $GF(p) \times GF(p)$ . We show that these families have good auto and cross correlation values in the sense that the peak auto correlation value maintain good ratio with off-peak values and we briefly describe their use in video.

**Keywords:** watermarks, auto-correlation, cross-correlation

## Problema de pertenencia en ideales de un dominio de polinomios

*Luis F. Cáceres*, Departamento de Ciencias Matemáticas, Universidad de Puerto Rico en Mayagüez.

Decimos que los ideales de un anillo  $R$  son detachables si uno puede decidir efectivamente si un elemento dado del anillo pertenece o no a un ideal finitamente generado dado de  $R$ . Usando el hecho que los ideales de  $Z[x]$  son detachables presentamos un procedimiento efectivo para encontrar una base mínima para un ideal  $A$  de  $Z[x]$ , a partir de un conjunto finito de generadores de  $A$ . Más aún, dada una base mínima para el ideal  $A$  de  $Z[x]$  es posible determinar efectivamente si un polinomio arbitrario  $f(x)$  de  $Z[x]$  está en  $A$ . De hecho, la dificultad computacional para determinar pertenencia en  $A$  está basada en encontrar su base mínima. Las ideas presentadas pueden ser generalizadas para anillos de polinomios sobre ideales principales.

**Palabras claves:** ideales, polinomios, enteros

## MSEIP-INTERactua: a student success network for STEM students in first-year mathematics courses

*Carmen Caiseda (et al.)*, Department of Natural Sciences and Mathematics, Inter American University of PR at Bayamón.

The MSEIP-INTERactua program developed a student success network at the Inter American University of PR in from October 2014 to October 2018. The student success network targeted first-year mathematics students that were in STEM programs. This comprehensive structure consisted in the use of the Social Network, educational hand-held technology with classroom connectivity, workshops for students and faculty, real-mathematics context, and student services that included a first-year counselor and tutoring. Retention and success rates of first-year courses will be presented as well as lessons learned in the multiple strategies implemented.

**Acknowledgements:** This program and research was sponsored by the U.S. Department of Education Minority Science and Engineering Improvement Program (MSEIP), award P120A140036. The work was conducted in collaboration with Javier Sierra, Gilda Díaz, Rafael R. Canales, Nilsa Toro, Joel Maldonado, and Abdel Valenzuela from the Inter American University at PR-Bayamón, and Omar Hernandez and Pascua Padró from the University of Puerto Rico at Río Piedras.

## La aplicación de funciones a la teoría de $\tau$ -factorizaciones

*José E. Calderón Gómez*, Department of Mathematics, University of Puerto Rico at Mayagüez.  
*Reyes M. Ortiz-Albino*, Department of Mathematics, University of Puerto Rico at Mayagüez.

Denotemos a  $D$  un dominio con integridad,  $U(D)$  el conjunto de elementos invertibles o unidades de  $D$  y  $D^\#$  el conjunto de elementos distintos de cero que no son unidades de  $D$ . La teoría de  $\tau$ -factorizaciones fue desarrollada por Anderson y Frazier en 2006, como una generalización de la teoría usual, añadiendo a la discusión una relación binaria sobre los elementos del dominio. Los autores consideraron relaciones simétricas sobre los elementos que no son cero ni unidades del dominio. A un producto de la forma  $a = \lambda a_1 \cdots a_n$ , donde  $\lambda \in U(D)$  y  $a_i \tau a_j$  para todo  $1 \leq i < j \leq n$  se le conoce como  $\tau$ -factorización. A los elementos  $a_i$  se les llama  $\tau$ -factores de  $a$  y  $a$  es un  $\tau$ -producto de estos. En este concepto también es posible obtener casos especiales de  $\tau$ -factorizaciones como las factorizaciones en átomos o las factorizaciones en elementos primales, entre otros. Este trabajo está centrado en el estudio de una relación en particular, la relación  $\tau_f$ , donde  $f$  es una relación entre dominios con integridad. La relación  $\tau_f$  se construye a partir de una relación binaria  $\tau$ , sobre  $D^\#$ , la relación se define como  $\tau_f = \{(x, y) : afx, afy \text{ y } (a, b) \in \tau\}$ . Para analizar esta nueva relación, se estudian que propiedades se pueden obtener a partir de la relación  $\tau$ . Se presentarán ejemplos claves para obtener una idea más clara de este nuevo concepto y algunos resultados.

**Keywords:**  $\tau$ -factorización, átomo, relación binaria

# Convolutional deep learning to predict physical characteristics from genomic data

*Heriberto Carbia Gutierrez*, Department of Mathematics, University of Puerto Rico at Río Piedras.

*Steven Van Belleghem*, Department of Biology, University of Puerto Rico at Río Piedras.

*Riccardo Papa*, Department of Biology, University of Puerto Rico at Río Piedras.

*Edusmildo Orozco*, Department of Computer Science, University of Puerto Rico at Río Piedras.

Imagine if we could know what a person looks like from a sample of DNA. Identifying the genomic changes that control morphological variation has major importance to studying genetic disease as well as understanding evolutionary change. Machine learning (ML) approaches have the power to significantly improve the identification of complex genomic variation that is associated with morphological variation. Over the past decade ML has revolutionized entire fields (i.e., speech recognition, natural language processing, image classification, bioinformatics), however, its application to problems in medical and evolutionary genetics is still in its early stages. Recent work has demonstrated that convolutional neural networks (CNNs) perform particularly well at handling genomic data.

In this talk, we will discuss applications of a convolutional deep learning framework to classify phenotypic characteristics from genotypes. These genotypes are the set of genetic variants responsible for variation in a particular trait. For this framework, we are using genomes from different color pattern variants of a group of butterflies (i.e., *Heliconius spp.*) to train the CNN. Sample inputs of genomic data are treated as images and fed to the model as multi-dimensional arrays. Each input image then goes through several layers of convolution in which smaller filters of learned weights slide through each image identifying and extracting local features and patterns to create an *output feature map*. These image-like feature maps are eventually flattened and fed into a densely connected classifier network whose output is the predicted class corresponding to a specific phenotype. Learning consists of adjusting the network weights such that the difference between true and predicted class is minimized.

To establish a baseline performance, results from the proposed convolutional framework are compared to results using the traditional multi-level perceptron (MLP) network and a special type of recurrent neural network (RNN) commonly used for sequential, time series data known as a long short-term memory (LSTM). As hypothesized, preliminary results show that the proposed CNN framework outperforms both MLP and LSTM models.

**Acknowledgements:** This research was sponsored by the EPSCoR RII Track-2 FEC project (award #1736026) of the University of Puerto Rico at Río Piedras.

**Keywords:** artificial neural networks, classification, convolution, deep learning, genotype, phenotype

# The anisotropic parabolic problem with Wentzell boundary conditions and variable exponents

*Carlos Carvajal-Ariza*, Department of Mathematics, University of Puerto Rico at Mayagüez.

*Alejandro Vélez-Santiago*, Department of Mathematics, University of Puerto Rico at Mayagüez.

Given  $\Omega \subset \mathbb{R}^N$  be a bounded domain with Lipschitz boundary  $\Gamma := \partial\Omega$ , for  $N \geq 3$ , and let  $p_i(\cdot)$ ,  $q_j(\cdot)$  be Lipschitz functions over  $\bar{\Omega}$  such that  $1 < p_i^- := \text{ess inf } p_i \leq p_i^+ := \text{ess sup } p_i < N$  and  $1 < q_j^- := \text{ess inf } q_j \leq q_j^+ := \text{ess sup } q_j < N - 1$  for each  $i \in \{1, \dots, N\}$  and  $j \in \{1, \dots, N - 1\}$ . Given  $\alpha \in L^\infty(\Omega)$  and  $\beta \in L^\infty(\Gamma)$  with  $\inf_{x \in \Omega} \alpha(x) \geq \alpha_0$  and  $\inf_{x \in \Gamma} \beta(x) \geq \beta_0$  for some constants  $\alpha_0, \beta_0 > 0$  and  $u_0 \in L^{r(\cdot)}(\Omega)$  for  $r \in \mathcal{P}(\Omega) := \{p : \Omega \rightarrow [1, \infty] \text{ measurable}\}$  such that  $1 \leq r^- \leq r^+ \leq \infty$ , we study the well-posedness of the quasi-linear parabolic equation with fully Wentzell boundary conditions, formally defined by

$$\begin{cases} u_t - \Delta_{\vec{p}(\cdot)} u + \alpha |u|^{p_M(\cdot)-2} u = f & \text{in } \Omega \times (0, \infty) \\ \sum_{i=1}^N |\partial_{x_i} u|^{p_i(\cdot)-2} \partial_{x_i} u \nu_i - \Delta_{\vec{q}, \Gamma} u + \beta |u|^{q_M(\cdot)-2} u = 0 & \text{on } \Gamma \times (0, \infty) \\ u(x, 0) = u_0 & \text{in } \Omega, \end{cases}$$

where

$$\Delta_{\vec{p}(\cdot)} u := \sum_{i=1}^N \partial_{x_i} \left( |\partial_{x_i} u|^{p_i(\cdot)-2} \partial_{x_i} u \right)$$

represents the (anisotropic)  $\vec{p}(\cdot)$ -Laplace operator and,

$$\Delta_{\vec{q}, \Gamma} u := \sum_{j=1}^{N-1} \partial_{\tau_j} \left( |\partial_{\tau_j} u|^{q_j(\cdot)-2} \partial_{\tau_j} u \right)$$

represents the (anisotropic) Laplace-Beltrami operator, where  $\partial_{\tau_j} u$  denotes the directional derivative of  $u$  along the tangential directions  $\tau_j$  at each point on  $\Gamma$ .

We show that the realization of the operator  $\Delta_{\vec{p}(\cdot)}$  with the above boundary conditions generates a (nonlinear) order-preserving submarkovian  $C_0$ -semigroup  $\{T(t)\}_{t \geq 0}$  of operator over  $L^{r(\cdot)}(\Omega)$ .

**Keywords:** anisotropic problems with variable exponents, Wentzell boundary conditions, order-preserving submarkovian nonlinear  $C_0$ -semigroup

## Learning good features to discriminate untagged bees in video using non-supervised learning

*Jeffrey Chan*, Department of Computer Science, University of Puerto Rico at Río Piedras.

*Iván F. Rodríguez*, Department of Mathematics, University of Puerto Rico at Río Piedras.

*Rémi Mégret*, Department of Computer Science, University of Puerto Rico at Río Piedras.

*José L. Agosto-Rivera*, Department of Biology, University of Puerto Rico at Río Piedras.

*Tugrul Giray*, Department of Biology, University of Puerto Rico at Río Piedras.

In this talk, we discuss a method to identify honeybees based on their appearance. To achieve it, we used deep learning and Siamese Network in a semi-supervised manner to learn an embedding that provides a numerical representation of their appearance. This embedding is trained to be similar for two images of the same bee, and dissimilar for images of different bees, using the contrastive loss. Training data can be obtained fully automatically by using automatic detection and tracking of the bees. The approach was evaluated for the identity verification task, where two images are compared to decide if they represent the same bee or not. The best model reached a performance of 0.94 in the area under the ROC curve. Our results show that this method has good performance to be used for re-identification task, where the appearance can be used to guide tracking in case of momentary disappearance.

**Acknowledgements:** This material is based upon work supported by the National Science Foundation under Grant No. 1707355 and 1633184.

**Keywords:** deep learning, unsupervised learning, Siamese neural network, insect behavior monitoring, identity recognition

## Computability-theoretic aspects of Ramsey's Theorem

*Peter Cholak*, Department of Mathematics, Notre Dame, IN.

Ramsey's Theorem for pairs and 2 colors says that for every 2-coloring,  $C$ , of pairs of natural numbers, there is an infinite set  $H$ , such that all pairs from  $H$  have the same constant color.  $H$  is called a *homogeneous* set for  $C$ . We will also consider Ramsey's Theorem in other settings. For example, we can be given countably many 1-coloring of natural numbers,  $\{c_i\}$ , and ask for a set  $C$  such that, for each  $i$ , all but finitely many elements of  $C$  have the same color with respect to  $c_i$ . Call  $C$  a *cohesive* set for  $\{c_i\}$ .

The question we will explore here is what can be computed from homogeneous sets. For example, given countable many 1-coloring  $\{c_i\}$ , is there a 2-coloring  $C$ , such that every homogeneous set for  $C$  computes a cohesive set for  $\{c_i\}$ ?

**Acknowledgements:** This work was partially supported by a grant from the Simons Foundation (#315283).

**Keywords:** Ramsey's Theorem, computable

## Analytical investigation of basic reproductive ratio in designing disease screen-treat programs

*Mayteé Cruz-Aponte*, Department of Mathematics-Physics, University of Puerto Rico at Cayey.  
*Özgür M. Araz*, Supply Chain Management and Analytics, College of Business Administration, University of Nebraska-Lincoln.

Chlamydia Trachomatis (CT) is one of the most frequently reported sexually transmitted diseases (STD) in the United States. Many cases of CT are either undetected or not reported. Recently, public health programs have been developed to reduce community-wide STD prevalence. These programs consist of screening and treatment of diseases and education for health behavior change. Public facilities of corrections play an important role in the effectiveness of these programs. However, due to lack of capacity and resources, public health programs using the correctional facilities raise concerns for policy-makers in terms of cost to society. In this paper, we present a mathematical model of CT transmission dynamics in two interacting populations using ordinary differential equations. Utilizing the dynamic model, we analyze the impact of jail based screen-treat programs and behavioral interventions on community health outcomes. Theoretical conditions for epidemic control are derived using the basic reproductive rate, and analytical investigations for considered interventions are presented. Informed by these analytical results, numerical simulation experiments are conducted for a case study population to evaluate the cost-benefits of interventions. Parametric sensitivity analyses are also performed to support practical insights.

**Keywords:** STD,  $\mathfrak{R}_0$ , simulation, public health, compartmental models, epidemiology, cost/benefit, Chlamydia Trachomatis

## Clustering methods to classify a broad range of species from their vocalization

*Omar Cruz Pantoja*, Department of Computer Science, University of Puerto Rico at Río Piedras.  
*Carlos J Corrada Bravo*, Department of Computer Science, University of Puerto Rico at Río Piedras.

Biodiversity monitoring is the process of tracking and determining changes in living organisms and the ecological complexes of which they are part of. It is important because it can provide information on environmental health, migration routes and population status. Traditionally biodiversity is assessed using methods that are generally costly, limited in space and time. Most fauna monitoring protocols require presence of experts in the field since data is often acquired through indirect cues such as vocalization. Such methods creates various problems. For starters, the requirement of an expert to identify species which itself can skew the data due to expert bias and the data recorded can not be replicated.

In order to solve replication and spatial content in time and space, systems with automated data collection can record audios 24 hours a day, every day of the year, across a variety of habitats. Yet automated data collection can collect an overwhelming amount of data and still requires an

expert to manually identify and classify all species present in the audio recordings. This task can still be tedious, extensive, time consuming and prone to classification mistakes. With the rise of the computational power and storage this task has been aided by the development of algorithms that automate the classification of presence or absence of a species through its vocalization. A limitation is that most of the algorithms developed target one or few species. We developed an algorithm that will be used to classify a broad range of species. It uses clustering methods to segment noise, identify regions of interest (ROIs) and classify using various machine learning techniques.

**Keywords:** acoustic detection, machine learning, clustering algorithms, classification of species

## The elliptic anisotropic problem with Wentzell boundary conditions and variable exponents

*Victor Díaz-Martínez*

Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

*Alejandro Vélez-Santiago*

Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

Let  $\Omega \subseteq \mathbb{R}^N$  be a bounded Lipschitz domain, for  $N \geq 3$ . We investigate the solvability and regularity of a class of quasi-linear elliptic equations involving the anisotropic  $\vec{p}(\cdot)$ -Laplace operator  $\Delta_{\vec{p}(\cdot)}$ , with nonhomogeneous anisotropic Wentzell boundary conditions

$$\sum_{i=1}^N \left| \frac{\partial u}{\partial x_i} \right|^{p_i(\cdot)-2} \frac{\partial u}{\partial x_i} \nu_i - \Delta_{\vec{q}(\cdot), \Gamma} u + \beta |u|^{q_M(\cdot)-2} u = g \quad \text{on } \Gamma := \partial\Omega,$$

for  $\beta \in L^\infty(\Gamma)^+$  with a positive lower bound, where  $\Delta_{\vec{q}(\cdot), \Gamma}$  denotes the anisotropic  $\vec{q}(\cdot)$ -Laplace-Beltrami operator and  $q_M(x) = \max\{q_1(x), \dots, q_{N-1}(x)\}$ . We show existence and uniqueness of weak solutions for the elliptic problem, and moreover, we prove that such solutions are globally bounded over  $\bar{\Omega}$ .

**Keywords:** anisotropic problems, variable exponents, Wentzell boundary conditions, weak solutions, a priori estimates.

## At most N-to-one functions over finite fields for new trapdoor functions

*Moises Delgado*, Department of Mathematics-Physics, University of Puerto Rico at Cayey.

We define a function over a finite field to be **at most  $n$ -to-one** (AMNTO) if every element in the range has at most  $n$  pre-images. The study of AMNTO functions and the search for infinite families of these functions may provide new methods for finding highly nonlinear functions and trapdoor functions, consequently new applications in cryptography. Numerous studies have been conducted on permutation polynomials since more than a hundred years ago, but almost no research has been focused on these functions. In this talk we will show some probable applications of AMNTO

functions and some families obtained during one year of research. A result obtained is a criterion for families of two-to-one binomial and trinomials  $x^s + ax^t$ ,  $x^s + ax^t + b$ , based on permutations polynomials. Particular examples are  $x^{13} + ax^2$ ,  $x^{18} + ax^3$  and  $x^{62} + ax + b$  over  $F_{16}$ ,  $F_{32}$  and  $F_{64}$  respectively. Using more sophisticated methods, we found the families of two-to-one trinomials and four-to-one quaternomials functions  $x^{2^k} + a^{2^k-1}x + a^{2^k}$  and  $x^{2^{2k}} + x^{2^k} + \alpha x + \beta$ , where  $a \in F_{2^n}$  and  $\alpha, \beta$  are sums of powers of  $a$ . These families are AMFTO on  $F_{2^n}$  when  $k, n$  are relatively prime. These methods included a combination of linear and quadratic functions as well as the use of APN functions.

**Keywords:** public key cryptography, trapdoor functions APN functions, gold functions, Kasami-Welch functions

### The Koch cube domain, a 3-dimensional Koch snowflake analogue

*Giovanni Ferrer Suárez*, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.  
*Alejandro Vélez-Santiago*, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

The Koch Snowflake has been a prominent object of study in the field of fractal analysis. We provide a construction via an iterated function scheme (IFS) of the Koch Cube domain  $K$ , a 3-dimensional analog of the Koch Snowflake. We then investigate fractal properties of the Koch Cube. We show that the boundary  $\partial K$  enjoys the open set condition (OSC), and thus obtain that  $\partial K$  has Hausdorff dimension  $s = \log_2(6)$  and Hausdorff measure  $0 < \mathcal{H}^s(\partial K) < \infty$ . Moreover,  $\mathcal{H}^s$  is an  $s$ -Ahlfors measure on  $\partial K$ . We further provide arbitrarily close upper and lower bounds for  $\mathcal{H}^s(\partial K)$  in the style of B. Jia (2005, 2007).

**Keywords:** fractal analysis, Koch cube, Hausdorff measure, Hausdorff dimension, bounds

### Classic functions, an analytic approach

*Javier Gómez-Calderón*, The Pennsylvania State University.

Complex numbers play an essential role in college algebra but are modestly used in the classic sequence of calculus. In this expository talk we provide a brief introduction to the complex-valued functions sine, cosine, exponential and logarithm. The main goal of this presentation is to provide an accessible reference to complex functions for undergraduate students who have completed calculus III.

## On cyclotomic subfields

*Javier Gómez-Calderón*, The Pennsylvania State University.

Let  $C_m$  denote the cyclotomic field generated by a primitive root of unity of order  $m$ . It is well known that the Galois group  $G(C_m/Q)$  is isomorphic to the group of units modulo  $m$ . Hence, the fixed field of any subgroup  $H$  of  $G(C_m/Q)$  is a Galois extension of  $Q$ . Given a divisor  $d$  of  $[C_m : Q]$ , we determine an algebraic integer  $\alpha$  generating a subfield  $F$  of degree  $d$  over  $Q$ . Furthermore, for the quadratic cases  $[C_m : Q(\alpha_1)]$  and  $[C_m : Q(\alpha_2)]$ , we also determine the minimal polynomials for  $\alpha_1$  and  $\alpha_2$ , and show that  $Q(\alpha_1) \cap B = Z[\alpha_1]$  and  $Q(\alpha_2) \cap B = Z[\alpha_2]$  where  $B$  denotes the ring of algebraic integers.

## Linear codes, graphs, and the projective special linear group $\text{PSL}_2(\mathbb{F}_q)$

*Anthony Gómez Fonseca*, Department of Mathematics, University of Puerto Rico at Río Piedras.  
*Heeralal Janwa*, Department of Mathematics, University of Puerto Rico at Río Piedras.

In this talk we discuss some results obtained in the master thesis' research of the first author under the supervision of Dr. Janwa. The projective special linear group  $G = \text{PSL}_2(\mathbb{F}_q)$  is the quotient group of the special linear group  $\text{SL}_2(\mathbb{F}_q)$  and its center. This group is used to construct a special type of graphs called Cayley graphs. The Cayley graph  $\Gamma(G, S) = (V, E)$  is the graph with vertex set  $V = G$  and edge set  $E = \{[v, w] \mid v^{-1}w \in S\}$ , where  $S$  is a symmetric subset of  $G$ . These graphs have been constructed and studied for some primes  $q$ . Once we construct the Cayley graph  $\Gamma(G, S) = (V, E)$ , we generate a bipartite graph  $\Gamma_B$ , called the edge-vertex incidence graph, with vertex set  $V \cup E$  and edge set  $\{[e, v] \in E \times V \mid v \text{ is an endpoint of } e\}$ .

The graph  $\Gamma_B$  is used to construct a linear code. A code  $\mathcal{C}$  of block length  $n$  is a subset of the vector space  $\mathbb{F}_q^n$ , but we limit our computations to the binary case. If  $\mathcal{C}$  is a subspace of  $\mathbb{F}_q^n$ , then we say that  $\mathcal{C}$  is a linear code. The elements of a code are called codewords. The code  $\mathcal{C}$  is constructed from a matrix  $H_{\mathcal{C}}$ , called the parity check matrix of  $\mathcal{C}$ , that is obtained from the graph  $\Gamma_B$ . Some properties of  $\mathcal{C}$  are studied, such as dimension, rate, and minimum distance. The minimum distance of a code is important because it determines the number of errors that it can correct. Given a code  $\mathcal{C}$ , some techniques can be used to obtain a subcode  $\mathcal{D} \subset \mathcal{C}$  with a higher minimum distance. Some examples will be consider.

**Acknowledgements:** Special thanks to the NASA Puerto Rico Space Grant Consortium for partially fund this research under the NASA Grant NNX15AI11H. Another special thanks goes to the High Performance Computing facility (HPCf), the University of Puerto Rico, and the Institutional Development Award (IDeA) INBRE Grant P20 GM103475.

**Keywords:** linear codes, minimum distance, cayley graphs, projective special linear group

## Linear complexity analysis of multidimensional periodic arrays

*Domingo Gómez-Pérez*, Faculty of Sciences, University of Cantabria, Spain.

*Andrew Tirkel*, Scientific Technology, Victoria, Australia.

Many researchers in different areas have studied multidimensional arrays and, in recent years, there has been wide interest in constructions of two-dimensional arrays with good correlation properties. One method to construct such arrays is the method of Composition, proposed by Tirkel, Osborne and Hall and generalized by Moreno and Tirkel, where a shift sequence and a column sequence are composed to generate a two dimensional array. Arrays constructed using this method have been proven to possess good correlation properties when the shift sequences have low Auto and cross hit numbers and where the column sequence is a cyclic shift of a pseudo noise sequence.

Linear complexity of an array is a measure of the order of a multidimensional linear recursion which generates the full array from a sub array.

In this talk, we discuss a recent approach to develop a theory for analyzing the linear complexity of general multidimensional periodic arrays. We also analyze arrays constructed using the method of composition and establish tight bounds for their linear complexity.

## An improvement to Chevalley's theorem with restricted variables

*Francis N. Castro*, Department of Mathematics, University of Puerto Rico at Río Piedras.

*Oscar E. González*, Department of Mathematics, University of Illinois at Urbana-Champaign, IL.

*Luis A. Medina*, Department of Mathematics, University of Puerto Rico at Río Piedras.

Chevalley's theorem states that a set of polynomials  $F_j(X_1, \dots, X_n)$  without constant terms over a finite field  $\mathbb{F}_q$  has a nontrivial common zero  $(a_1, \dots, a_n) \in \mathbb{F}_q^n$  if the number of variables exceeds the sum of the degrees of the polynomials. Recently Schauz and Brink gave an extension of Chevalley's theorem to polynomials with variables belonging to arbitrary non-empty subsets of a finite field. Here we apply the ground field method to Schauz-Brink's theorem. We obtain an improvement that is significant when the degree of the polynomials is large compared to the weight of the degree of the polynomials.

**Acknowledgements:** The second author was partially supported by a GAANN fellowship (#P200A150319, Department of Education) and by the Alfred P. Sloan Foundation's MPHD Program, awarded in 2017.

**Keywords:** solvability of systems of polynomials over finite fields; Chevalley's theorem

## Fundamental solutions for discrete dynamical systems involving the fractional Laplacian

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*Valentin Keyantuo*, Department of Mathematics, University of Puerto Rico, Río Piedras.

*Carlos Lizama*, Department of Mathematics, Universidad de Santiago de Chile.

*Mahamadi Warma*, Department of Mathematics, University of Puerto Rico, Río Piedras.

In this talk we prove representation results for solutions of a time-fractional differential equation involving the discrete fractional Laplace operator in terms of generalized Wright functions. Such equations arise in the modeling of many physical systems, for example chain processes in chemistry and radioactivity. Our focus is in the problem:

$$\mathbb{D}_t^\beta u(n, t) = -(-\Delta_d)^\alpha u(n, t) + g(n, t),$$

where  $0 < \beta \leq 2$ ,  $0 < \alpha \leq 1$ ,  $n \in \mathbb{Z}$ ,  $(-\Delta_d)^\alpha$  is the discrete fractional Laplacian and  $\mathbb{D}_t^\beta$  is the Caputo fractional derivative of order  $\beta$ . Also is presented important special cases as consequences of the representations obtained, such as heat and wave discrete equation.

**Acknowledgement:** The author was supported by CONICYT under grant Doctorado Nacional Chile 2016 NO:21160120.

**Keywords:** Caputo fractional derivative; discrete fractional Laplacian; fundamental solutions; Wright and Mittag-Leffler functions

## A problem of Ambrosetti-Prodi type for the anisotropic Laplace operator

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*Alejandro Vélez-Santiago*, Department of Mathematics, University of Puerto Rico at Mayagüez.

We investigate the realization of a very new class of nonlinear boundary value problems of Ambrosetti-Prodi type involving the variable exponent anisotropic  $\vec{p}(\cdot)$ -Laplace operator, and Neumann boundary conditions. Using a priori estimates, regularity theory, a sub-supersolution method, and the Leray-Schauder degree theory, we obtain a necessary condition for the non-existence of solutions (in the weak sense), the existence of at least one minimal solution, and the existence of at least two distinct solutions. Moreover, we establish fine global regularity results for weak solutions of the Neumann problem of Ambrosetti-Prodi type over large classes of non-smooth domains.

**Keywords:** anisotropic problems with variable exponents, Neumann boundary conditions, weak solutions, a priori estimates, global regularity, sub-supersolution method, Leray-Schauder degree theory

## On the characterization of $\tau_{(n)}$ -atoms

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*Reyes M. Ortiz-Albino*, Department of Mathematics, University of Puerto Rico at Mayagüez.

In a 2011 paper, Anderson and Frazier defined the concept of a  $\tau_{(n)}$ -factorization, where  $\tau_{(n)}$  is a restriction of the modulo  $n$  equivalence relation. These relations have been studied mostly for small values of  $n$ . One of these problems is finding  $\tau_{(n)}$ -irreducible elements or  $\tau_{(n)}$ -atoms in order to characterize elements that have a  $\tau_{(n)}$ -factorization in  $\tau_{(n)}$ -atoms. The  $\tau_{(n)}$ -atoms are well known for  $n = 0, 1, 2, 3, 4, 5, 6, 8, 10, 12$ . However, the problem of determining the  $\tau_{(n)}$ -atoms becomes much more difficult for larger  $n$ . In this research, we present an easy way to construct families of  $\tau_{(n)}$ -atoms, where  $n$  is a safe prime associated with a Germain prime.

**Acknowledgements:** This research, conducted at the University of Puerto Rico at Mayagüez, is supported by the Puerto Rico Louis Stokes Alliance For Minority Participation and by the Department of Mathematical Sciences of the University of Puerto Rico at Mayagüez.

**Keywords:**  $\tau_{(n)}$ -factorization,  $\tau_{(n)}$ -atom, Germain prime

## Cuantificación de la ventaja cognitiva de estudiar límites de funciones utilizando infinitésimos y Cálculo no Estándar con estudiantes del curso Métodos Cuantitativos II para Administración de Empresas

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*Carlos A. Lezama*, Co-Founder and CEO of AZapp INC.

En estudios anteriores se ha demostrado la eficacia del método de enseñanza de límites de funciones utilizando infinitesimales para estudiantes del curso de Calculo I en la facultad de Ciencias Naturales. El objetivo de la investigación que se lleva a cabo, es determinar si existe ventaja didáctica cuando se enseña el concepto de límite de una función real, utilizando el lenguaje de los números infinitésimos a estudiantes del curso de Métodos Cuantitativos para Administración de Empresas II de la Facultad de Administración de Empresas.

El propósito del estudio es determinar si las destrezas en el cálculo de límites para los estudiantes, mejora si se utiliza el cálculo infinitesimal para su enseñanza. Tal circunstancia haría que la noción de límite de una función, sea más evidente, de forma tal que los estudiantes entiendan mejor este concepto y puedan determinar con más facilidad límites de funciones.

La eficacia de la enseñanza se mide con el instrumento: Prueba de aprovechamiento de límites. La investigación se instrumenta con estudiantes del curso de Métodos Cuantitativos para Administración de Empresas II (MECU 3032), que ofrece el Instituto de Estadística y Sistema Computarizado de Información de la Facultad de Administración de Empresas de la Universidad de Puerto Rico, Recinto de Río Piedras y a quienes enseño este curso. Soy profesora, adscrita a dicha facultad.

Para recopilar y validar información se utilizarán exámenes desarrollados en dos modalidades:

Calculo tradicional o estándar y Calculo infinitesimal.

Para llevar a cabo el análisis de los datos y determinar los hallazgos de investigación se utilizarán medidas estadísticas para la respuesta a la pregunta de investigación: ¿existe una ventaja didáctica en enseñar el concepto de límite de una función real, utilizando el lenguaje de los números infinitesimos a estudiantes del curso de Métodos Cuantitativos para Administración de Empresas II de la Facultad de Administración de Empresas de la universidad de Puerto Rico, Recinto de Río Piedras?

**Palabras claves:** ímite, función, infinitesimal, aprendizaje, enseñanza, cálculo

## Public key cryptosystems based on AG codes suitable for post- quantum PKC

*Heeralal Janwa*, Department of Mathematics, University of Puerto Rico at Río Piedras.

McEliece public key cryptosystem (PKC) has received increasing attention in recent years as an alternative to other PKCs because it is one of the very few candidates for Post-Quantum PKCs.

Janwa and Moreno proposed two types of PKCs in 1996 using AG codes: Type I based on the subfield subcodes of AG codes, and Type II based on AG Goppa codes ( ISIT-95 (invited), DCC-96). Our Type I PKCs are still secure but our Type II PKCs have been shown to have weak keys.

We have developed a general framework of public/private key ensembles for PKCs based on Algebraic Geometry, and discuss their practicality and crypt- analysis [J-(Geocrypt 2009),J-(ICERM2015), J-2016], where we have proposed a thrid type of PKCs based on AG codes. We present results on their feasibility, practicality, and security.

After massive worldwide collaborative computing efforts, the ensemble of original McEliece PKCs based on binary irreducible classical Goppa codes with parameters [1024, 524,101] has been shown to be insecure. The original McEliece PKC has exponential key lengths, and thus is not easily scalable. Alternatives, such as shortening or list-decoding binary irreducible Goppa codes have been proposed as PKCs. However, we demonstrate how our Type I and Type III PKCs provide better alternatives with superior parameters. Our PKCs are similar to those of McEliecs. Our Type II class of PKCs has been shown to have weak keys, even after already excluding the genus 0 case (i.e., those corresponding the generalized RS codes)- as they were known to be insecure by Sidelnikov-Shestakov. A refinement of Sidelnikov-Shestakov has demonstrated that under some assumptions, our Type II PKCs based on insecure for codes from low genus curves and for curves with special properties. We show why these are the weak keys and discuss how to avoid other potentially weak keys, and how to choose stronger ones.

We also outline several deep algebraic geometric problems that our three classes of PKCs based on AG codes lead to (moduli space of curves, bounds on exponential sums over curves, existence of curves with many rational points, existence of certain divisors, etc.). We will give some partial results and suggest some open problems. These algebraic geometric problems have also applications to several other areas in mathematics, computer science, and communication sciences. For example in construction of Ramanujan graphs, expander graphs, ECC, random number generators, APN functions, hash functions, and sequence designs.

**Keywords:** post-quantum, cryptoSystems, algebraic geometric codes, McEliece public key cryptosystems based on subfield subsides of AG codes

## Criptografía simplicial

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Los complejos simpliciales tienen importantes aplicaciones en la topología. Entre los complejos simpliciales más sencillos se encuentran los grafos no dirigidos. Al desarrollar el concepto de operaciones sobre complejos simpliciales y aplicar una de estas operaciones a una clase de grafos, llamados grafos de Erdős-Rényi, se producen complejos simpliciales que exhiben características aleatorias. A partir de eso, se sugiere cómo se puede usar esta construcción para encriptar un mensaje.

## Comparison of the frequencies of numbers free of small primes, and squares or cubes, with their probabilities inferred from Eulers identity for the harmonic series

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We count the frequencies of numbers free of small primes, and also of numbers free of squares and cubes, in random samples generated with the computer in different ranges, and compare them with the theoretical probabilities that are expressed in terms of the reciprocal of harmonic series.

As is well known, the numbers of primes in a range can be theoretically estimated by several expressions that follow from Eulers identity between Eulers product and the harmonic series. If we limit us to observe numbers free of small primes, probabilities given this way are exact and we can compare them with actual counts in random samples of numbers. Less known, Eulers identity with integer power larger than one gives the probability for numbers free of squares, cubes and other powers, and we can also check them empirically. As the harmonic series of power larger than one are convergent, the restriction to small primes is now less important. Our calculations agree with the theory and illustrate the meaning behind the harmonic series and the zeta function.

We can go further and interpret Eulers products for any real power as the probabilities for not having multiples of powers of primes near a randomly selected natural number. This way we can give also a full interpretation of the zeta function on the reals larger than one.

**Acknowledgements:** This work was motivated by the Creative Project for the masters degree of Roberto Romero-Nieves.

**Keywords:** prime, square free, power free, Eulers product, harmonic series, number theory

## Los grafos de $\tau_{(n)}$ -divisores $\tau_{(n)}$ -irreducibles

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La teoría de  $\tau_{(n)}$ -grafos o los grafos de  $\tau_{(n)}$ -divisores  $\tau_{(n)}$ -irreducibles, surge como una posible aplicación de la teoría de  $\tau_{(n)}$ -factorizaciones. La misma se desarrolla extendiendo a la teoría de factorizaciones generalizadas y el trabajo de Coykendall y Maney sobre los grafos de divisores irreducibles sobre dominios. Aunque la idea fue propuesta desde sus comienzos, no fue hasta el 2010 que Lucena, bajo la supervisión de Ortiz, desarrolló la idea y se presentó en varias conferencias. Luego en el 2013, Mooney publicó unos primeros resultados que conectan las propiedades de dominios con las propiedades de grafos, tal y como lo hizo Coykendall. Por ejemplo, un dominio integral  $D$  es un UFD si y solo si el  $\tau$ -grafo simple de cualquier elemento distinto de cero y que no sea unidad es un grafo completo.

Presentamos algunas propiedades y características generales de los  $\tau_{(2)}$ -grafos. Consideramos propiedades de maximidad e isomorfismos entre ellos, estudiando las propiedades de los grados de los vértices en los  $\tau_{(2)}$ -grafos y formalizamos algunos patrones que los mismos poseen. Presentamos algunas propiedades combinatorias que los  $\tau_{(2)}$ -grafos tienen, condiciones necesarias para que un grafo simple sea un  $\tau_{(2)}$ -grafo y por último algunas generalizaciones de estos resultados.

**Acknowledgements:** This research, conducted at the University of Puerto Rico at Mayagüez, is supported by the Puerto Rico Louis Stokes Alliance For Minority Participation and by the Department of Mathematical Sciences of the University of Puerto Rico at Mayagüez.

**Keywords:**  $\tau_{(n)}$ -divisor,  $\tau_{(n)}$ -irreducible,  $\tau_{(n)}$ -grafo

## The 5-cube cut number problem; a short proof for a basic lemma

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*Rafael Arce-Nazario*, Department of Computer Science, University of Puerto Rico at Río Piedras.

The hypercube cut number  $S(d)$  is the minimum number of hyperplanes in the  $d$ -dimensional Euclidean space  $\mathbb{R}^d$  that slice all the edges of the  $d$ -cube. The problem originally was posed by P. O'Neil in 1971. B. Grünbaum, V. Klee, M. Saks and Z. Füredi have raised the problem in different times. In 2000, Sohler and Ziegler obtained a computational solution to the 5-cube problem. However finding a short proof for the problem, independent of computer computations, remains to be a challenging problem. We present a short proof for the result presented by Emamy-Urbe-Tomassini in Hypercube 2002 based on the Tomassini's Thesis. This proof is only a few pages that is substantially shorter than the original proof of 60 pages.

**Keywords:** hypercube, slicing hyperplanes, polytopes

## Modeling coffee fruit infestation by the coffee berry borer

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*Amitabha Bose*, Department of Mathematical Sciences, New Jersey Institute of Technology.

*Paul Bayman*, Department of Biology, University of Puerto Rico at Río Piedras.

The coffee berry borer (CBB) is the most damaging insect pest of coffee worldwide; understanding the dynamics of its reproduction is essential for pest management. A mathematical model of the infestation progress of the coffee fruit by the CBB during several coffee seasons is formulated. By using a system of ordinary differential equations the model represents the interaction between the CBB and the coffee fruit populations. Coffee harvesting is also included in the model. A one-dimensional map is used for tracking the population dynamics subject to certain coffee harvesting percentages over several seasons. The map possesses two fixed points, a stable one corresponding to eradication of the infestation, and an unstable one that separates initial CBB populations that can be eradicated from those that lead to total infestation. Stability analysis of the map's fixed points shows that a specific level of CBB infestation could be eliminated over multiple seasons of coffee harvesting. However, the percentage of coffee harvesting required is determined by the level of CBB infestation at the beginning of the first season and in some cases such a percentage cannot be achieved, which leads to total infestation.

**Keywords:** biological control, coffee berry borer, mathematical modeling, one-dimensional map, pest control management

## Permutation polynomials over finite fields

*Ariane Masuda*, Department of Mathematics, New York City College of Technology, CUNY.

A polynomial  $f(x)$  over a finite field  $\mathbb{F}_q$  is called a *permutation polynomial* over  $\mathbb{F}_q$  if the induced mapping  $f : \mathbb{F}_q \rightarrow \mathbb{F}_q$  is a bijection. The research on permutation polynomials has been intensified over the past decades due to their applications in many areas such as coding theory and cryptography. In this talk I will present several families of permutation polynomials, and discuss the techniques utilized to find them.

**Acknowledgements:** The author received support from an award jointly funded by The Professional Staff Congress and The City University of New York, and from a Minority Science and Engineering Improvement Program.

**Keywords:** permutation polynomials, finite fields

## Working with Francis Castro

*H. F. Mattson*, Syracuse University.

I will discuss my work as a co-author with Francis Castro, without going into much technical detail, on these two papers:

“Correction to Divisibility properties for covering radius of certain cyclic codes IEEE Trans. Inform. Theory, vol. 52 (2006) pp. 1798-1799 (with Moreno, O. and Castro, F. N.)

“Divisibility of exponential sums via elementary methods, Journal of Number Theory, vol. 130 (2010) pp. 1520-1536 (with Francis N. Castro, Hugues Randriam, and Ivelisse Rubio)

I will also explain how Francis continues today to influence my work.

## Puerto Rico crime cap, a visualization and analysis of a geographical dataset

*Ollantay Medina Huaman*, Department of Mathematics, University of Puerto Rico at Humacao.

Data visualization is an important tool of the Data Science workflow. Big geographical datasets require an especial treatment to achieve a dynamic and fluid map visualization. In this work, we use a set of high-level packages for data visualization, called PyViz, to produce a map of crimes across Puerto Rico.

The geographical dataset contains around 250,000 records, with crimes registered between 2012 and 2016. The main map visualization provides controls that enable different types of interaction with the data. Secondary dependent dynamic views show summarized data or results from other analysis techniques.

**Keywords:** data science, visualization, geographical data, crime, PyViz

## Infinite energy cavitating solutions: stabilization of the energy functional and its numerical approximation

*Pablo V. Negrón-Marrero*, Department of Mathematics, University of Puerto Rico at Humacao.  
*Jeyabal Sivaloganathan*, Department of Mathematical Sciences, University of Bath, Bath UK.

We study radial solutions of the equations of isotropic compressible elasticity when the Dirichlet part of the stored energy function grows like  $\|\nabla \mathbf{u}\|^n$ . Here  $\mathbf{u}$  is the deformation of the body and  $n$  is the space dimension. It follows that cavitating solutions for the corresponding energy functional have infinite energy. We introduce a modified energy functional for which cavitating minimizers have finite energy. Moreover, the Euler–Lagrange equations for the modified energy functional are identical to those for the original problem except for the boundary condition at the inner cavity. This boundary condition now states that a certain modified Cauchy stress function (which is

monotone increasing on solutions) has to vanish at the inner cavity. Most of the results for finite energy cavitating solutions follow through for the modified functional, like existence of minimizers, satisfaction of the EL-equations for such minimizers, and the existence of a critical boundary displacement for cavitation. We also discuss a numerical scheme for computing these cavitating solutions via solutions over punctured balls. We show the convergence of this numerical scheme and give some numerical examples.

**Keywords:** nonlinear elasticity, cavitation, infinite energy solutions

## Stochastic maximum principle for controlled switching diffusions with conditional mean-field

*Son L. Nguyen*, Department of Mathematics, University of Puerto Rico at Río Piedras.

*Dung T. Nguyen*, Department of Mathematics, HCM City University of Technology, Vietnam.

*George Yin*, Department of Mathematics, Wayne State University.

In this talk we discuss a maximum principle for switching diffusions with mean-field interactions. The motivation stems from a wide range of applications in networked systems in which large-scale systems are encountered and in which mean-field interactions are involved. Because of the complexity due to the switching, little has been done for the study of the associate control problems with mean-field interactions. The main ingredient of this work is the use of conditional mean-field, which is distinct from the existing literature. Using the maximum principle, optimal controls of linear quadratic Gaussian controls with mean-field interactions for switching diffusions are carried out. Numerical examples are also provided for demonstration.

**Acknowledgements:** This research was supported by a seed fund of Department of Mathematics at University of Puerto Rico, Río Piedras campus.

**Keywords:** maximum principle, mean-field interaction, switching diffusions

## Explorando la Ciencia de Cómputos para Puerto Rico: algunas experiencias y resultados del proyecto ECS4PR\*

*Edusmildo Orozco (et al.)*, Department of Computer Science, University of Puerto Rico at Río Piedras.

ECS, *Exploring Computer Science* (<http://www.exploringcs.org/>), consta de un programa de desarrollo profesional y de un currículo introductorio a la ciencia de cómputos diseñado para escuelas secundarias. La misión de ECS es ampliar la participación en computación de forma equitativa y usando el enfoque de aprendizaje basado en la indagación.

El proyecto *Exploring Computer Science for Puerto Rico* (ECS4PR) tiene como meta principal establecer una alianza de investigación participativa en educación para apoyar el programa de

desarrollo profesional de ECS y la implantación piloto de su currículo con adaptaciones lingüísticas y culturales relevantes a Puerto Rico, en varias escuelas públicas.

En esta charla presentaremos: las preguntas y los instrumentos desarrollados para la investigación; algunos resultados preliminares; una descripción del sistema de apoyo a maestros; los retos, las oportunidades y los planes a mediano plazo para lograr un impacto a mayor escala en la Isla. Además, presentaremos algunas de las experiencias que han tenido las maestras y maestros en la implantación del currículo en algunas escuelas públicas de Puerto Rico.

**Acknowledgements:** Este material está basado en trabajo apoyado por la Fundación Nacional de las Ciencias bajo el Grant No. 1738577. Este trabajo se realizó en colaboración con Michelle Borrero, Patricia Ordóñez, Joseph Carroll-Miranda, Luis López, Agustín Corchado, Gerrian Houser y Eliud Gerena de la Universidad de Puerto Rico en Río Piedras, y Nilda Delgado, Damaris Arroyo, Noemí Vázquez, Elizabeth Ríos y Ernesto J. Díaz del Departamento de Educación de Puerto Rico.

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**Keywords:** Exploring Computer Science, ECS, desarrollo profesional, ciencia de cómputos

## Fast FPGA implementations of elliptic curve point multiplication for a family of special finite fields

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*Dorothy Bollman*, Department of Mathematics, University of Puerto Rico at Mayagüez.

We consider elliptic curves over a certain family of special finite fields of approximately the same size of the recommended bit lengths 192, 224 and 256 for elliptic curve cryptography and we propose a fast FPGA implementation of point multiplication operation for these curves. Special fields are of the form  $GF(p^3)$  defined by a irreducible binomial  $x^3 - 2$  modulo  $p = 2^a - 2^b - 1$  and with a claimed levels of security equal to 96, 112 and 128 bits. We synthesized and simulated our implementation on Virtex-5 and Virtex-6 FPGAs families. Comparing our running times with the best times currently found in the literature, we found that our times are up to 18 times faster.

**Keywords:** elliptic curve, point multiplication, FPGA, optimal extension field

## La teoría de las $\tau$ -factorizaciones

Reyes M. Ortiz-Albino, Department of Mathematics, University of Puerto Rico at Mayagüez.

La noción de las  $\tau$ -factorizaciones o  $\tau$ -productos sobre dominios con integridad surge como un tipo de factorizaciones generalizadas de finidas por Anderson y Frazier, en el 2006. Esta nueva noción sirve como base para el estudio de las factorizaciones de una estructura. Entre los resultados principales de Anderson y Frazier, el mas que resalta es la caracterización de tres tipos de relaciones  $\tau$  que garantizan propiedades de factorizaciones. Tales propiedades son naturales en los productos usuales y hacen que propiedades de la estructura de factorizaciones del dominio con integridad se hereden a las  $\tau$ -factorizaciones. Se proveeran ejemplos de distintos tipos de estas relaciones y los resultados principales. Las ideas básicas servirán de preámbulo para las charlas de Jose Calderón, Christian López y André Hernández.

**Keywords:** generalized factorizations, factorizations

## Non Uniform Dependence for Euler Equations in Besov Spaces

José Pastrana, Department of Mathematics, University of Notre Dame.

The use of differential equations to model physical phenomena traces back to the work of Sir Isaac Newton and his second law of motion, which for constant mass states:  $F = m \cdot a$ . We apply this formula to the motion of an ideal (non - viscous) and incompressible fluid, say with mass  $m = 1$ , with velocity field  $v = v(x(t), t)$ , where  $x(t)$  denotes its position at time  $t$  and with acceleration  $a = \partial_t v$ . Using the Chain Rule and the assumption that the force is conservative (physicists convention) we get

$$-\nabla p = F = m \cdot a = 1 \cdot \partial_t \left[ v(x(t), t) \right] = \sum_j \frac{\partial v}{\partial x_j} \cdot \frac{\partial x_j}{\partial t} + \partial_t v$$

commonly written as  $\partial_t v + (v \cdot \nabla)v + \nabla p = 0$  and called Euler equations of fluid dynamics; here  $p$  is the pressure of the fluid. The incompressibility condition translates to  $\operatorname{div} v := \sum_j \partial_j v_j = 0$  and  $v_0(x) := v(x, 0)$  denotes the initial configuration of the fluid.

Ever since Hadamard defines the notion of local well posedness for models of physical phenomena (the existence of a unique solution where the solution's behavior changes continuously with respect to the initial conditions) many have shown interest in the regularity properties of the data to solution map,  $v_0 \rightarrow v$ , for various evolution equations. In the case of Euler equations this goes back to the papers of Kato and Ponce.

We make use of a construction due to Himonas and Misiolek and the approximate solutions technique to show that such map it is not uniformly continuous for a family of generalized functions with special regularity properties known as Besov spaces. This family include to say some, the classes of Sobolev and Hölder continuous functions.

**Keywords:** fluid dynamics, data to solution map, approximate solutions technique

## Existence and Properties of Maximal Metrics

Samuel Pérez-Ayala, University of Notre Dame.

Let  $M^n$  be a closed (compact with no boundary)  $n$ -dimensional Riemannian manifold endowed with a conformal class  $[g]_1$  of unit volume metrics. The concept of *Conformal Spectrum* was introduced by Colbois & Soufi in 2003 to talk about the supremum of  $\lambda_k(\Delta_g)$  within the conformal class  $[g]_1$ , where  $\lambda_k(\Delta_g)$  denotes the  $k$ -th eigenvalue corresponding to the Laplace Beltrami operator  $\Delta_g$  on a surface  $M^2$ . The existence of a metric  $\tilde{g} \in [g]_1$  achieving such supremum have been found to be tightly related to the existence of a very special type of harmonic maps. I will discuss an analogous problem on a four manifold  $M^4$  for a fourth order elliptic operator  $P_4$  called the Paneitz Operator. This operator was discovered by Stephen Paneitz in 1983, and it satisfies similar conformal properties as the Laplace operator does in dimension 2.

## Redefine Statistical Significance

Luis R. Pericchi Guerra, Department of Mathematics, University of Puerto Rico at Río Piedras.

In an article in "Nature, Human Behavior", published September 01, 2017, over 70 Scientists and Statisticians across the globe and across disciplines proposed to change the threshold for significance from  $\alpha = 0.05$  (Fisher's Scale) to  $\alpha = 0.005$  (Alternative Scale). So far, January 25 2019, the article has received 381 citations, which anticipates that it is going to have a powerful influence in Science as a whole. We review the motivations of the proposal, prominently the **Crisis of Reproducibility** in Science and suggest improvements on it.

## Binary quantum codes from the Hermitian curve

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Fernando L. Piñero, Department of Mathematics, University of Puerto Rico at Ponce.

BCH codes provide for a simple code construction over a small alphabet, while still maintaining the structure of the corresponding Reed–Solomon supercodes. By studying the parameters of BCH–like codes from curves covered by the Hermitian curve over  $\mathbf{F}_{q^2}$  we can find good binary and quaternary subcodes which can outperform the corresponding BCH code. As the Algebraic Geometry codes from the Hermitian curve are self–orthogonal, we also study the question of finding good quantum codes from subfield subcodes of the corresponding AG codes.

**Keywords:** Reed–Solomon codes, Hermitian codes, algebraic geometry codes, binary Codes, quantum error correction

## Optimal locally recoverable codes of distance 5 from affine cartesian codes

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*Ryan Coatney*, University of Arizona.

*Venkatesan Guruswami*, Carnegie Mellon University.

*Hiram López*, Clemson University.

*Fernando L. Piñero*, Department of Mathematics, University of Puerto Rico at Ponce.

Locally repairable codes (LRCs) have recently received significant attention as a method of designing data storage systems robust to server failure. Optimal LRCs offer the best trade-off between minimum distance and locality. For optimal LRC's of minimum distance greater than or equal to 5, the block length is bounded by a polynomial function of the alphabet size. In this presentation, we give a construction of an optimal LRC of minimum distance 5 using Affine Cartesian codes.

**Keywords:** optimal LRC, Reed–Solomon codes, affine cartesian codes

## Music: Art, Physics and Mathematics, a practical example of interdisciplinary teaching and learning

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*Héctor Jiménez*, Department of Physics, University of Puerto Rico at Mayagüez.

*Arturo Portnoy*, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

Three professors, a music theorist, a physicist, and a mathematician offer an innovative and interdisciplinary course exploring music and its relationship with mathematics and science. The course was first offered in the Fall of 2017, with a very enthusiastic reception by the 80 students who took it, even with the interruption and chaos caused by hurricane Maria in Puerto Rico. We are offering it again for the Spring semester of 2019 to 90 students. The results for the course, both the surveys and the final projects, indicated that students in the course were able to strengthen their integrative skills.

The course is designed for a general audience with no prerequisites, and as a consequence tends to a very heterogeneous population. Every meeting includes a 30 minutes period of music education, where musical theory and practice is presented and built from scratch, followed by a 50 minutes of mathematics or science (mostly physics) alternating every week.

Readings are assigned for every meeting and there is a short electronic quiz every morning about the assignment to encourage reading. During the music section, playing with a recorder or singing is an integral part of learning about notation, timing, melody, harmony, scales, chords, etc.

In the physics section topics such as waves, amplitude, frequency, resonance, are explored both in theoretical discussions and demonstrations. The physics of the auditory apparatus, the perception of sound and music, the construction of musical instruments are some of the other topics discussed.

During the mathematics section Pythagorean musical theory is introduced, as well as the idea of musical scales and their tunings or temperaments. Patterns in music, for example in rhythm

and harmony, analysis and synthesis of sound and music, and digital information and music are also discussed.

Integration of all disciplines is emphasized in the discussion of more philosophical ideas, such as: the difference between noise and music, beauty in the context of music, consonance, dissonance, and harmony.

All three professors are present in every lecture, and active discussion with the students and all three instructors is encouraged. Agreement and disagreement among the instructors is evidenced in real time through these discussions and witnessed by the students, encouraging the idea that civilized dissension is possible and even desirable.

Writing and team work is encouraged with a midterm team essay on a topic previously approved by the teaching team, and a practical team based final project (also previously approved) such as the construction of an instrument, the composition and performance of a musical piece, or the practical exploration and presentation of a suitable topic. The final project involves a 10 minute presentation in front of the class. The integration of music, science and mathematics is paramount in both midterm and final.

This course is part of a campus-wide initiative: Expanding the Humanities in the University of Puerto Rico, Mayaguez Campus for designing interdisciplinary courses that integrate the humanities and STEM fields in introductory general education courses, using writing and communication skills, as well as team work, to further support and deepen the students integrative skills.

## **Clasificación de Microfotografías de Epidermis de Hojas utilizando Redes Neuronales artificiales**

*Elio Ramos*, Department of Mathematics, University of Puerto Rico at Humacao.

Presentamos los resultados de un estudio de clasificación de microfotografías de hojas utilizando redes neuronales y aprendizaje profundo (deep learning). Las hojas fueron colectadas en un bosque seco en la Isla de Mona localizada entre República Dominicana y Puerto Rico. Las imágenes consisten de una muestra de 69 clases (especies) de plantas en donde se puede apreciar la epidermis del lado abaxial (inferior) de la hoja a escala microscópica (200X). Para mejorar el rendimiento de la clasificación se utilizó la técnica de aumento de datos (data augmentation) extendiendo de esta manera la cantidad de muestras del conjunto de entrenamiento. Se construyó un modelo de clasificación basado en una arquitectura de red de convolución utilizando el paquete Keras/Tensorflow de Python. Resultados preliminares para las 69 clases (especies) de plantas indican una precisión de clasificación de 98%.

**Keywords:** aprendizaje automático, clasificación, aprendizaje profundo, procesamiento de imágenes

## A geometric proof for the existence of travelling wave solutions for a model of epidemic wounds

Joaquin Rivera, Department of Mathematics, University of Puerto Rico at Humacao.

A wound is classified as an epidermal wound when the epidermis is injured, but the dermis and the flesh beneath the wound are not damaged. Although, some of the cellular and molecular processes that lead to epidemic healing are well understood, there are still a significant number of them that remain unknown. For this reason, we decided to apply mathematical models to elucidate additional information about these healing processes. During the talk, we will discuss a mathematical model describing the healing of epidermal wounds. The model consists of a pair of partial differential equations of reaction-diffusion coupled, one for the density of epidermal cells and the other for the concentration of a chemical activation or inhibition of mitosis. We will use geometric singular perturbation theory to show the existence of traveling wave solutions. Moreover, numerical results will be presented to emphasize the roles of the different parameters of the model and the clinical implications of this mathematical model.

**Keywords:** traveling wave solutions, slow and fast manifolds

## On an infinite family of satellite knots and its knot polynomials

Mónica M. Robles Fontán, Department of Mathematics, University of Puerto Rico at Río Piedras.  
Iván Cardona, Department of Mathematics, University of Puerto Rico at Río Piedras.

Knot theory has been recurring as an area of mathematical research since the 19th century but its popularity has grown and continues to grow since the 1980's. Being the classification of knots the main aim of knot theory, we use knot polynomials to classify an infinite family of satellite knots built using trivial knots in  $\mathbb{S}^3$  as pattern knots and nontrivial companion knots,  $C$ . In particular, we show that any satellite knot ( $SK$ ) in the infinite family will have Alexander polynomial  $\Delta_{SK}(t) = \Delta_C(t^{|r|+1})$ , where  $|r| + 1$  is the number of trivial knots that are generated through the construction of the satellite knot. Furthermore, we give an expression for the Kauffman bracket and the writhe of the satellite knots in study which will lead us to an expression of the Jones polynomial.

**Acknowledgements:** This research work was funded by the National Science Foundation through the Puerto Rico Louis Stokes Alliance for Minority Participation (PR-LSAMP) as part of the Bridge to the Doctorate Program.

**Keywords:** satellite knot, Alexander polynomial, Kauffman bracket, writhe, Jones polynomial

# Automatic monitoring of the foraging behaviour of tagged and untagged honey bees

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*Edgar Acuña*, Department of Mathematics, University of Puerto Rico at Mayagüez.

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In this work we use a deep learning method for detection, localization and tracking of honeybee body parts from video to study honey bee foraging behavior patterns including timing of the trips and the presence of not presence of pollen when returning.

The proposed approach integrates several techniques to provide the complete vision of the foraging activity at the ramp of the Hive. First using the 5 key-points (head, tip of the abdomen, thorax, left antenna and right antenna) detected by the deep learning module and the subsequent tracking to find trajectories that enters and exits the colony. These trajectories are used then to detect the events. Also using the orientation provided by the detection of the head-tail limb, a compensated cropped image of the bees is evaluated on a pollen detector model that retrieves whether the bee is bringing pollen.

Training was performed using 300 frames extracted randomly from videos of the last week of June at different times. Each frame of 2048x1600 pixels with fully labeled individuals. Validation was performed with 100 frames also extracted randomly but belonging to different days. Once trained, the model was used to analyze all full week from the 21st of June 2018 until the 28th of June 2018 from 8:00 Am in the morning until 5:00 Pm in the afternoon. In order to evaluate performance of the event detection we compared to manually labeled events of tagged bees provided on all the videos from 21 st of June.

In honeybees, tracking and studying behaviour in natural conditions is crucial to enhance our understanding of their complex social activities, their biological rhythms and find new explanations for the risk conditions that can be detrimental to hive health. The proposed system opens new possibilities for large scale analysis of video for long periods of time.

**Acknowledgements:** This material is based upon work supported by the National Science Foundation under Grants No. 1633164 and 1633184. I. F. Rodriguez acknowledges support from Janelia Visitor Program. T. Giray acknowledges support from Puerto Rico Science and Technology Trust (PRSTRT; 2016-00161, 2017-00164). The authors would like to thank Stephanie Feliciano and Janpierre Aleman who helped with the acquisition at the UPR Agricultural Station of Gurabo of the videos used in the generation of the dataset and Jeffrey Chan who helped with the annotation of the videos.

**Keywords:** honeybee behavior analysis, computer vision, deep learning, pose estimation, pollen detection

## Modeling ultrafine particulate matter through motor vehicles emissions

*Michelle N. Rosado Pérez*, Bioengineering Graduate Program, University of Puerto Rico at Mayagüez.  
*Karen Ríos Soto*, Department of Mathematics, University of Puerto Rico at Mayagüez.

The inhalation of air pollutants can be very dangerous to human health causing nausea, vision damage, fatigue, elevated blood pressure, central nervous system injuries and even death. Although there are several types of air pollutants, this research studies particulate matter, specifically the ultrafine particulate matter. Ultrafine particles (UFP) are particles with aerodynamic diameters of less than  $0.1 \mu\text{m}$  that originate mainly from the combustion of motor vehicles. Because of their small sizes, UFPs can get access to the gas exchange regions of the lungs and move to other parts of the body through the blood. Asthmatic individuals are more likely to be affected by ultrafine particles, since UFPs can exacerbate their episodes of asthma. UFPs have not been well studied and there is no regulation in the United States that controls their emissions, which is alarming since they are considered the most dangerous type of particulate matter. In this talk, the Air Pollutants Models will be introduced. The model consist of a reaction diffusion-advection partial differential equation that models how the pollutants growth and disperse in the environment. Through the analysis of the model it was possibly to identify which conditions are necessary to control the pollutants emissions in the environment, as well as to identify the parameters that have more influence in changing the pollutants concentrations. The use of filters in the motor vehicles and the switching of more eco-friendly ways of transportation was suggested as a way to reduce the UFPs emissions.

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**Keywords:** particulate matter, ultrafine particles, motor vehicles

## Extension of the covering method to any finite field

*Ivelisse Rubio*, Department of Computer Science, University of Puerto Rico at Río Piedras.

The covering method for polynomials over the prime field  $F_p$  was introduced by Castro and Rubio (2014, 2015) as an elementary method to compute exact  $p$ -divisibility of exponential sums. The divisibility of exponential sums has been used to characterize and prove properties in coding theory, cryptography and solvability of polynomial equations. In general, algebraic methods to estimate the  $p$ -divisibility of exponential sums are non-elementary. In the papers mentioned above, the proofs only require elementary methods. Here we extend the covering method to any field  $F_q$ , keeping the simplicity of the results, but the proofs require tools that are no longer elementary.

**Keywords:** exponential sums,  $p$ -divisibility, covering method

## Implementación de criptografía simplicial

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*Guarionex Salivia*, Computer and Information Science Department, Minnesota State University.

*Jonathan Hardwick*, Computer and Information Science Department, Minnesota State University.

Se introduce el uso de complejos simpliciales para procesos de encriptación. Grafos Erdős-Rényi con pocos vértices y aristas, permiten la transmisión de mensajes con bajo costo computacional. Luego de aplicar operaciones en los grafos, obtenemos complejos simpliciales cuya descripción codificada en bits se puede utilizar para encriptar un mensaje haciendo uso del algoritmo conocido como one-time-pad. Verificamos empíricamente que la descripción del complejo simplicial resultante es una secuencia de bits aleatorio que se pueda usar como un “one-time-pad”.

## Recursions associated to trapezoid, symmetric and rotation symmetric functions over Galois fields

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*Luis A. Medina*, Department of Mathematics, University of Puerto Rico at Río Piedras.

*Leonid Brehsner Sepúlveda*, Department of Mathematics, University of Puerto Rico at Río Piedras.

Rotation symmetric Boolean functions are invariant under circular translation of indices. These functions have very rich cryptographic properties and have been used in different cryptosystems. Recently, Thomas Cusick proved that exponential sums of rotation symmetric Boolean functions satisfy homogeneous linear recurrences with integer coefficients. In this work, a generalization of this result is proved over any Galois field. That is, exponential sums over Galois fields of rotation symmetric polynomials satisfy linear recurrences with integer coefficients. In the particular case of  $\mathbb{F}_2$ , an elementary method is used to obtain explicit recurrences for exponential sums of some of these functions. The concept of trapezoid Boolean function is also introduced and it is showed that the linear recurrences that exponential sums of trapezoid Boolean functions satisfy are the same as the ones satisfied by exponential sums of the corresponding rotations symmetric Boolean functions. Finally, it is proved that exponential sums of trapezoid and symmetric polynomials also satisfy linear recurrences with integer coefficients over any Galois field  $\mathbb{F}_q$ . Moreover, the Discrete Fourier Transform matrix and some Complex Hadamard matrices appear as examples in some of our explicit formulas of these recurrences.

**Keywords:** rotation functions, trapezoid functions, symmetric polynomials, exponential sums, recurrences

## Value distribution of elementary symmetric polynomials and their perturbations over finite fields

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*Leonid Brehmsner Sepúlveda*, Department of Mathematics, University of Puerto Rico at Río Piedras.

*César A. Serna-Rapello*, Department of Mathematics, University of Puerto Rico at Río Piedras.

In this talk we establish the asymptotic behavior of generating functions related to the exponential sum over finite fields of elementary symmetric functions and their perturbations. This asymptotic behavior allows us to calculate the probability generating function of the probability that the elementary symmetric polynomial of degree  $k$  and its perturbations returns  $\beta \in \mathbb{F}_q$ , where  $\mathbb{F}_q$  represents the field of  $q$  elements.

**Keywords:** exponential sums, symmetric functions, perturbations, value distribution

## Movimientos migratorios en Puerto Rico de 1850 a 1984

*José O. Sotero-Esteve*, Departamento de Matemáticas, Universidad de Puerto Rico en Humacao.

La literatura sobre la historia de Puerto Rico del siglo XX identifica tres olas migratorias saliendo de la rularía hacia el área metropolitana de San Juan: durante las primeras décadas del siglo XX, a partir de la década de 1940 y durante los años sesenta. Los datos del censo agrupados por regiones es la fuente más común entre los trabajos consultados para este estudio que mencionan su fuente de datos. No encontramos en la literatura un estudio basado en conteo de individuos. En nuestro estudio se consideró como área metropolitana los pueblos de San Juan, Río Piedras, Bayamón y Carolina. De los datos sobre más de dos millones de defunciones disponibles en el portal [datapr.gov](http://datapr.gov) se seleccionó 840,509 personas que no nacieron en esos pueblos y que vivieron 30 años o más. Entre los hallazgos más llamativos se encontró un máximo de migración cerca del 1895 con picos menores cercanos a los años 1929 y 1958.

Aparte de los hallazgos mencionados arriba este estudio presenta otros de interés histórico a una resolución de pueblo que es mayor a la encontrada comúnmente. Además se prueba la viabilidad del uso de datos demográficos crudos para estudios históricos mediante el uso de métodos y herramientas de la ciencia de datos para su análisis y presentación. Finalmente, el procedimiento seguido puede servir de base para proyectos en cursos de bancos de datos o de ciencia de datos.

**Reconocimientos:** El autor agradece la subvención de equipo y tiempo para investigación del proyecto UPRH-Penn Partnership for Education and Research in Materials (NSF Grant NO. DMR-1523463).

## Dynamics of Nonlinear Wave Propagation in Hyperbolic Metamaterials

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*Brooke Pitre*, Department of Mathematics, Lamar University, Beaumont, TX.

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In this work the propagation dynamics of nonlinear waves in Hyperbolic metamaterials is explored in detail by mean of planar dynamical system theory. The proposed model that governs such dynamics is a Nonlinear Schrödinger Equation (NLSE) equipped with spatio-temporal dispersion features, self-steeping effects, and the Raman scattering effect. Exact traveling wave solution will be provided in terms of Jacobi elliptic functions. The relation between solitary waves solution and Jacobi elliptic periodic solution will be also stressed through numerical and analytical results in view of the influence of spatiotemporal dispersion, along with the discussion of corresponding qualitative analysis.

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**Keywords:** Schrödinger equation; traveling waves; integrability; bifurcation; metamaterials

## Solvability over $C(\overline{\Omega})$ of the parabolic anisotropic Robin problem with variable exponents

*Maria-Magdalena Boureanu*, Department of Applied Mathematics, University of Craiova, Romania.

*Alejandro Vélez-Santiago*, Department of Mathematics, University of Puerto Rico at Mayagüez.

We develop fine regularity results for a time dependent Robin problem of anisotropic structure and variable exponents. In particular, we show that the part related to the anisotropic  $\vec{p}(\cdot)$ -Laplace operator  $\Delta_{\vec{p}(\cdot)}$  generates a strongly continuous nonlinear semigroup over the space of continuous functions  $C(\overline{\Omega})$ , which establishes the well-possessedness of the parabolic problem over  $C(\overline{\Omega})$ . In addition, Hölder-type ultracontractivity results are provided.

**Keywords:** Anisotropic problems with variable exponents, Robin boundary conditions, nonlinear semigroups, ultracontractivity property

## 5 Afiches / Posters

(In alphabetical order using the last name of the presenter.)

### **Matrices with good cross-correlation and autocorrelation for watermarking 2d digital images**

*Cesar F. Bolaños Revelo*, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

*Dorothy Bollman*, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

Easy access via the internet to multimedia, and in particular to digital images, poses a copyright problem for the authors. Various alternatives have been proposed to solve this problem. Among these are watermarks, which introduce secure information that guarantees that it is the property of the author and that can be detected only by the author, based on the method used. Watermarks embedded in the spacial domain are accomplished by directly manipulating the pixels, making the watermark secure and impossible to detect by all but the owner. The matrix of cross-correlations between the pixel matrix of the watermarked image and the watermark allow us to verify and recuperate the watermark by means of the position of the maximum element, called the peak value. If there is a big difference between the peak value and the other elements, the watermark has good autocorrelation and cross-correlation values, an important characteristic for secure embedding of the watermark.

In this presentation we study some 2D watermarks embedded in the spacial domain, using matrices with good auto- and cross-correlation. These matrices include Legendre matrices and matrices generated by cyclic shifts of a column consisting of a sequence such as a Legendre sequence, an M-sequence, or a Hall sequence. We also discuss the results of implementing the embedding of these watermarks into grayscale digital images.

**Keywords:** cross-correlation, autocorrelation, watermarks, space domain

### **DevSecOps: continuous integration meets containers security**

*Jordan A. Caraballo-Vega*, Department of Mathematics, University of Puerto Rico at Humacao.

*Jasaun J. Neff*, High Performance Computing Branch, NASA Goddard Space Flight Center.

Continuous Integration (CI) is a software development technique where developers integrate code into one shared repository on a frequent basis. CI key principles reside in revision control, build automation, and automated testing; seeking to decrease debugging time while increasing development productivity. While in an ideal world CI itself seems to be the perfect approach to boost the productivity of software teams, its exposure to the network makes it vulnerable to various kinds of malicious attacks.

Based on these concerns, the NASA Center for Climate Simulation (NCCS) has refined the idea of enhancing CI with security techniques, while isolating jobs into containerized agents. The aim

of this project is to research and build a reinforced DevSecOps infrastructure able to significantly reduce security risks and increasing the ability to monitor each build cycle while increasing development speed. The core of the infrastructure resides in a six nodes cluster which includes Bamboo, Jira, and Docker services; together with a set of static, compliance, and dynamic scanners.

Three pipelines to simulate interactions between software developers, system administrators, and the combination of both were produced. The developers pipeline was intended to validate multiple software projects in search of software vulnerabilities and compliance, while the system administrators pipeline integrates and validates standalone container images as the representation of production-like operating systems. The combination of both seeks to integrate software projects and approved containers to simulate the behavior and interaction of the provided software services on a production-like system.

It can be concluded that a CI infrastructure needs to be reinforced with continuous assurance techniques to prevent additional security risks that the execution and integration of software could bring into play. These techniques hold the potential to speed up the development of software and could also serve as a testing environment for system administrators. Future work will include the integration of new tools and its wider deployment.

**Acknowledgements:** This research was sponsored by the NASA Minority University Research and Education Project.

**Keywords:** software development, continuous integration, security, systems development life cycle

## Mathematical Modeling with Human Mobility of the ZIKA virus in Puerto Rico

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In this poster we introduce a new and enhanced ZIKA virus model that incorporates human mobility in order to study the impact of mobility in the spread of the ZIKA virus in Puerto Rico. Vertical transmission of the aedes aegypti is also incorporated, together with mosquito demographics. A system of twelve nonlinear ordinary differential equations is defined and solved numerically using a simultaneous 4<sup>th</sup> and 5<sup>th</sup> order Runge-Kutta method. The numerical solution of the model using MATLAB will be presented and discussed. The reproduction number  $R_0$  is studied using the next generation matrix approach. This model is based on the classical disease dynamics SIR model introduced by Kermack and McKendrick in 1927.

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**Keywords:** zika, SIR, reproduction number, nonlinear ode

## Analysis of social factors influencing heroin addiction

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The spread of drug use throughout a community can be represented through epidemiological models typically found in studies on the dynamics of infectious disease. These mathematical models are based on the idea that, similar to infectious diseases, drug use spreads through interactions between individuals. Previous models of drug use don't take into consideration the preferences an individual may have in their interactions. We propose a compartmental model for heroin use that accounts for preferred mixing and as well as the element of choice in seeking or avoiding individuals based on their drug use habits.

We ran numerical solutions to identify outcomes that are particularly sensitive to changes in certain parameter values. Furthermore, we identified certain social interactions that may be critical in determining the level of heroin use throughout a community. We observed how different preferences affected these sensitivities and final values of the endemic equilibria.

We found that in some cases having the sober class and the addict class have a net avoidance of each other may be beneficial in minimizing heroin use, while in other cases, increasing social contacts between former users and current dependent users may result in a smaller proportion of the population suffering from heroin use disorder.

**Keywords:** opioid, addiction, differential equations, probability, preferred mixing

## A Mathematical Modeling Analysis for Evaluating Cost-Effective Treatment Strategies for the Control of HSV-2 Infection in the U.S.

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Infection of Herpes Simplex Virus type 2 (HSV-2) is a lifelong sexually transmitted disease. According to the Center for Disease Control (CDC), 11.9% of the United States (U.S.) population were infected with HSV-2 in 2015-2016. The HSV-2 pathogen establishes latent infections in neural cells and can reactivate causing lesions later in life, a strategy that increases pathogenicity and allows the virus to evade the immune system. HSV-2 infections are treated by Acyclovir in the non-constitutional stage, marked by genital skin lesions and ulcers. However, patients in the constitutional stage expressing mild symptoms, such as fever, itching, and painful urination, remain difficult to detect and untreated. In this work, we develop and analyze a mathematical model to study the transmission and control of HSV-2 among the U.S. population between the ages of 15-49. The goals of this

work are to compare the cost effectiveness of (i) treating HSV-2 infected in both constitutional and non-constitutional stages to (ii) the current conventional treatment protocol for treating patients in the non-constitutional stage. Our results distinguish model parameter regimes where each of the two treatment strategies optimizes the available resources and consequently gives the long-term reduced cost associated with each treatment and incidence. The public health cost of HSV-2 with the newly implemented treatment would increase from 16 to 20 billion. However, early treatment reduces HSV-2 incidence by 38% yearly and the reproduction number decreases by 40% from 2.5, which is based on current conventional treatment protocol. In conclusion, the proposed treatment strategy could be effective in controlling the transmission of HSV-2 at the population-level.

**Acknowledgements:** This research was conducted as part of 2018 MTBI at the Simon A. Levin Mathematical, Computational and Modeling Sciences Center (MCMSC) at Arizona State University (ASU). This project has been partially supported by grants from the National Science Foundation (NSF - Grant MPS-DMS-1263374 and NSF - Grant DMS-1757968), the National Security Agency (NSA - Grant H98230-J8-1-0005), the Alfred P. Sloan Foundation, the Office of the President of ASU, and the Office of the Provost of ASU. This work was done in collaboration with Luis Almonte-Vega (UPR Mayagüez), Ligia Luna-Jarrin (Instituto Nacional de Investigación en Salud Pública, Quito, Ecuador), Joel Martinez (Southwestern University, Georgetown, Texas), Jordy Rodriguez-Rincon (Universidad Nacional de Colombia, Bogotá), Rohan Patil, Baltazar Espinoza and Anuj Mubayi (Arizona State University), Anarina Murillo (University of Alabama, Birmingham), and Leon Arriola (University of Wisconsin, Whitewater).

**Keywords:** latent infections, incidence, Acyclovyr

## Binomials of the form $x^m[x^{\frac{q-3}{2}} + A]$ that do not produce permutations

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Permutations are used in important applications such as coding theory and cryptography. We study binomials of the form  $x^m(x^{\frac{q-3}{2}} + A)$  over finite fields. In this research project we prove that this class of binomials never permutes a finite field when: (a)  $A = \alpha^k$  where  $k$  is odd and  $\alpha$  is a primitive root of the field, or (b)  $A = \alpha^k$  where  $k$  is even,  $\alpha$  is a primitive root of the field and  $q = 4h + 1$ , and  $h \in \mathbb{N}$ . We are left with the case  $A = \alpha^k$  where  $k$  is even and  $q = 4h + 3$  and  $h \in \mathbb{N}$ . We conjecture that this case never permutes a finite field.

**Keywords:** permutations, permutation binomials, finite fields

## Phenotype response to higher order mutation interactions, a spectral analysis approach

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Mutations pave the way for evolution, and, with the growing amount of genomics data, we need efficient and effective ways to analyze interactions of gene mutations. We present a spectral analysis approach to orthogonally decompose a genomic data set and analyze higher order interactions between mutations. This approach offers insight into the effects that certain gene mutation groups have on hemoglobin while accounting for redundant information. This method can be extended to analyze other phenotypic responses as long as the dataset possesses certain properties.

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**Keywords:** spectral analysis, epistasis detection, genomics

## Recognition of Fanning Bees from Video using Convolutional Neural Networks

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The objective of this work is to study and test convolutional neural networks (CNN) to develop classifiers to identify the bee behavior of fanning, given video of the bees. We follow the work of I. Rodríguez et al. [1], where they built a video capture system for the entrance of a bee hive and developed new state-of-the-art pollen classification methods. Fanning bees stand on the entrance ramp of their hive and beat their wings, producing a current of air that serves the hive. We used the same dataset to develop the annotation and supervised classification systems. The dataset was created by extracting the individual images of the bees, with their respective fanning/not-fanning labels. The orientation of the bees was compensated to ensure in all image samples that the bee is facing upwards. With this information, the image dataset was built fixing the size of the cropping rectangle to 350x320 pixels, such that the annotated thorax position appears centered and that the bee is fully visible. The annotation consists in the position of the bees thorax, its orientation angle, and the identification of a fanning bee. Through this work in progress, we have currently annotated 155 video segments of fanning and not-fanning bees. Stratified split is used to create the training, testing and validation datasets, to make sure each bee is present in only one of the subsets. Compared to pollen recognition, the biggest challenge with fanning recognition is that fanning is a timed behavior instead of a static behavior. That is, we are working with video segments instead of individual images. For fanning, the bee will have entered the ramp area and could commence

the behavior at any time. The bee may or may not conclude the behavior before leaving the video capture area. Therefore, we must recognize the start and end of the behavior and not lose track of the bee. Additional challenges include capturing the moving wings given their speed and the overlap of other bees, and not having clear cues such as color. For this work we present preliminary results using shallow CNNs of 1 or 2 layer models. Performance of these models on the first dataset are encouraging, as they range from 70% to 98% accuracy. As future work, larger annotation will be pursued to refine these evaluations, and we will test deep CNN architectures using the Tensorflow Object Detection API.

**Acknowledgements:** This material is based upon work supported by the National Science Foundation under Grants No. 1633164 and 1633184. T. Giray acknowledges support from Puerto Rico Science and Technology Trust (PRSTRT; 2016-00161, 2017-00164). Edward Latorre acknowledges support from PRLSAMP. This work was done in collaboration with Kelvin López from the UPR at Arecibo, and Ivan F. Rodríguez, Matías Rosner, Rémi Mégret, Tugrul Giray and José L. Agosto from the UPR at Río Piedras.

**Keywords:** computer vision, convolutional neural networks, image classification, honeybees

## Deriving near-surfaces bulk densities of near-earth asteroids using radar observations

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Meter-scale particles can appear in radar image as brightness speckles, ubiquitously distributed on the surface of asteroids in the inner Solar System. They are indicative of the presence of centimeter-to-decimeter scale particles (“cobble”). Their spatial and size distribution give us indications of the geological evolution and the outcome of the collision event history. The abundance of cobbles can be suggestive of whether the asteroid is monolithic or a rubble pile. We use radar observations by the Arecibo Observatory Planetary Radar Program. Arecibo radar system transmits a powerful circularly polarized signal using a frequency of 2380 MHz (wavelength of 12.6 cm) and receives the echo in the opposite circular (OC) polarization and same circular (SC) polarization. The quasi-specular reflection from a layer of fine-grained regolith and diffuse scattering by and between the wavelength-scale particles compose the OC polarized part of the echo. The intensity and the polarization are suggestive of the physical properties of the targets near-surface. Here we investigate the physical properties of four Near-Earth Asteroids (NEAs), specifically analyzing 1998 WT<sub>24</sub>, 1998 CS<sub>1</sub>, 1950 DA, and 2006 AM<sub>4</sub>. Wavelength-scale particle distribution and near-surface bulk density of the underlying layer of sub-centimeter-scale particles of NEAs are derived from the radar albedos in the SC and OC polarization states. We derive the Fresnel reflection coefficient ( $R_F$ ) using a linear least squares fit to separate the diffuse-scattering part from the quasi-specular part of OC radar albedo ( $\sigma_{OC}$ ), to calculate the electric permittivity ( $\epsilon$ ). In order to derive the near-surface bulk densities we related both laboratory measurements of rocky lunar regolith and state-of-the-art methodology of interpreting electromagnetic radar scattering. We present preliminary results of bulk densities of  $2.6 \pm 0.54 \text{ g cm}^{-3}$ ,  $2.54 \pm 0.2 \text{ g cm}^{-3}$ ,  $3.0 \pm 1.7 \text{ g cm}^{-3}$ , and  $2.86 \pm 0.1 \text{ g cm}^{-3}$  for

1998WT<sub>24</sub>, 1998 CS<sub>1</sub>, 1950 DA, and 2006 AM<sub>4</sub>, respectively.

**Acknowledgements:** Thanks to Puerto Rico NASA Space Grant Consortium fellowship grant no. NNX15AI11H by supporting part of this research. Also, thanks to NASA's Near-Earth Object Observations Program through grant no. NNX13AQ46G awarded to Universities Space Research Association for the Arecibo Planetary Radar Program and, after April 2018, through grant no. 80NSSC18K1098 awarded to the University of Central Florida by support this research. The Arecibo Observatory is a facility of the National Science Foundation operated under cooperative agreement by University of Central Florida, Yang Enterprises, Inc., and Universidad Metropolitana.

## **Un modelo matemático para evaluar los efectos de la herbivoría de la polilla (*Oidaematophorus Espeletiae*) sobre el frailejón (*Espeletiae Grandiflora*)**

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La *Espeletia Grandiflora* Humb. and Bonpl 1809 o conocida popularmente con frailejón es una planta endémica de los Andes Orientales de Colombia que se reproducen únicamente en los ecosistemas de páramo a una altura que oscila entre los 9500 ft a 13450 ft, ésta planta cumple una tarea muy importante en el ecosistema colombiano ya que su objetivo principal es almacenar el agua que capta de la lluvia y de las neblinas que se forman en las alturas de las cordilleras andinas, esta especie regula la producción de agua que gota a gota llena los caudales de pequeñas quebradas que luego se convertirán en lagunas y grandes ríos. En el año 2012 se encontró que la *Espeletia Grandiflora* esta siendo atacada por una polilla, disminuyendo así su capacidad de almacenamiento de agua lo que podría ser un problema muy grave a largo plazo ya que esta abastece de agua a gran parte de la población de Bogotá Colombia. Por lo tanto, este trabajo propone un modelo matemático discreto con estructura de edad y con competencia entre etapas para estudiar la dinámica de la interacción entre la polilla y el Frailejón.

Primero se propone un modelo de ecuaciones de diferencia no lineal para estudiar el crecimiento poblacional del frailejón y se establecen las condiciones umbrales para el crecimiento natural de la población de frailejones a partir de la condición de estabilidad de los equilibrios. Se hacen simulaciones numéricas del modelo determinístico y se hacen simulaciones estocásticas para evaluar cómo afecta la variabilidad del clima en condiciones de páramo en el crecimiento poblacional del frailejón para observar el comportamiento general del sistema a lo largo de varios años. Finalmente se le añade al modelo del frailejón la interacción con la polilla.

**Palabras claves:** modelo de estructura de edad, modelo de competencia, estabilidad de equilibrios, modelo discreto, modelo huésped-parásito

## On the number of solutions to systems of polynomial equations with multivariate polynomials over finite fields as coefficients

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One of the fundamental problems in mathematics is whether or not it is possible to obtain a solution for a system of polynomial equations. In An Improvement of a Theorem of Carlitz, Castro, Moreno and Rubio generalize the results of Moreno-Morenos Theorem that gives a sufficient condition for a power of prime to divide the number of common zeros of a system of multivariate polynomials. This generalization regarded the coefficients of said polynomials to be uni-variate polynomials over a finite field instead of plain elements of the finite field. Consequently, this result led to improve a theorem of Carlitz, for the estimation of the number of variables needed so that a system of polynomial equations with coefficients that are uni-variate polynomials over a finite field can have non-trivial solutions. It has been suggested that these results can be further generalized to systems of polynomials with multivariate polynomials over finite fields as coefficients. In this work, we will generalize the results of Castro, Moreno and Rubio to systems of polynomials with multivariate polynomials as coefficients. Our plan is to follow the strategy of using the reduction to the ground field technique.

**Acknowledgements:** This research is sponsored by the Bridge to the Doctorate (BDP) fellowship, provided by the Puerto Rico Louis Stokes Alliance for Minority Participation (PR-LSAMP) and the Computational Sciences Department at UPR Rio Piedras.

**Keywords:** finite fields, system of equations, polynomials, multivariate

### Introduction to $f$ -factorization

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The  $f$ -factorization was motivated by the notion of a  $\tau$ -factorization, defined by Anderson and Frazier in 2011. We define  $a \cdot b$  to be an  $f$ -product if and only if  $a, b$  are non-zero non-unit elements such that  $f(a) = b$  with  $f$  as a function, instead of the symmetric relations used by Anderson and Frazier. This new concept does not coincide with the one introduced by them, except for the case of the identity function. Factorizations with respect to the identity function is very well understood. However, that is not the case for  $f$  as an arbitrary function. We have focused on evaluating several examples using polynomials and the existence of nontrivial  $f$ -products in general. We present interesting results for polynomials over the integers that allows us to identify what elements have an  $f$ -factorization. Also, those results were extended to integral domains and some of them create algebraic structures when  $f$  is a homomorphism. In addition, there are some cases where an element have  $f$ -factorizations of just certain lengths.

**Acknowledgements:** This research is under the PRLSAMP undergraduate research program with the supervision and collaboration of Dr. Reyes M. Ortíz-Albino.

**Keywords:** factorizations, products, algebraic structures

## Some reduced linear modular systems and their structures

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Let  $p$  be a prime,  $\mathbb{Z}_p$  be the integers modulus  $p$ , and  $\mathbb{Z}_p^n$  be the  $n$ -dimensional vector space over  $\mathbb{Z}_p$ . A *linear modular system* (LMS) is a pair  $(\mathbb{Z}_p^n, S)$ , where  $S$  is a linear function (i.e.,  $S$  is an  $n \times n$  matrix over  $\mathbb{Z}_p$ ). Let  $L_S = (\mathbb{Z}_p^n, S)$  be a non-singular LMS and  $M$  be a non-singular matrix over  $\mathbb{Z}_p$  with the property that  $MS = SM$ . It is known that the action of  $M$  over the cycles of  $L_S$  induces a new system called a *reduced linear modular system* (RLMS). Let  $m$  be a positive integer,  $A$  be a  $2 \times 2$  matrix over the ring  $\mathbb{Z}_m$ , and  $\mathbf{x} = (x, y)$  be an element of  $\mathbb{Z}_m^2$ . In this work we deal with the solution to the homogeneous system  $A\mathbf{x} = \mathbf{0} \pmod{m}$ , its relation to certain RLMSs, and the Chinese Remainder Theorem. The solution to this problem is important for optimizing a multidimensional fast Fourier transform of prime-edge length with linear symmetries in its inputs.

**Acknowledgements:** Part of this project is supported by Claude Shannon Scholarship (NSF-DUE 1356474).

**Keywords:** reduce linear modular systems, Chinese Remainder Theorem

## Análisis de datos de simuladores de conducción aplicando modelos de regresión funcional

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En ingeniería de transportación se hacen estudios en simuladores que buscan analizar el comportamiento de los conductores dado las características del conductor y diferentes escenarios basados en los elementos de diseño de las carreteras. Usualmente se recolecta información de velocidad o desplazamiento en intervalos cortos de tiempo (cada 0.016 segundos), lo cual implica que el número de observaciones en estos estudios es extremadamente grande. Actualmente, las prácticas de análisis de estos datos involucran i) dividir el dominio del experimento o recorrido en zonas o segmentos, ii) reducir los datos de cada segmento a una medida descriptiva, usualmente la media, y iii) aplicar técnicas de análisis de datos convencionales tales como modelos mixtos a los datos reducidos. Sin embargo, estas prácticas son ineficientes ya que usan una versión reducida de los datos. Por lo tanto, este trabajo propone modelos de regresión funcionales con respuesta funcional y covariables

escalares para analizar los datos de simuladores de conducción.

Un conjunto de datos reales del simulador de conducción de la Universidad de Puerto Rico en Mayagez (UPRM) es analizado usando el enfoque propuesto. Una de las ventajas de los modelos propuestos es que el comportamiento de los conductores se puede caracterizar completamente a través del tiempo usando curvas suavizadas y sus derivadas. Después de un proceso de selección de modelos, se concluyó que un modelo razonable para determinar las variables significativamente relacionadas a la velocidad al conducir es un modelo funcional lineal mixto con intercepto aleatorio funcional, efectos suaves del tiempo del día, ancho del carril y efecto escalar de la velocidad límite. Los modelos se ajustaron con la función `pffr(.)` disponible en la librería `refund` del programa estadístico R.

**Palabras claves:** modelo mixto, datos funcionales, modelo funcional lineal mixto, simulador de conducción

## Predicción de lluvia en Puerto Rico: un enfoque con redes neuronales artificiales

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Una red neuronal artificial (ANN, por sus siglas en inglés) es un modelo de computación que simula el cerebro siguiendo varias etapas de entrenamiento, validación y prueba para aprender y modelar patrones complejos de los datos y hacer predicciones. Actualmente las ANNs están siendo usadas en aplicaciones tales como reconocimiento de voz e imágenes, finanzas, diagnósticos médicos, predicción de lluvias, entre otras muchas.

El agua es un recurso natural vital para la vida humana. El evento de sequía ocurrido en Puerto Rico en el año 2015 demostró cuán importante es la lluvia, su almacenamiento, su distribución y el uso racional del agua para el desarrollo de toda actividad humana.

Motivados por la sequía del año 2015, en este trabajo exploramos un enfoque con ANNs para predecir las lluvias cuyo propósito es que pueda ser usado como herramienta en la planificación y manejo de las sequías en Puerto Rico.

**Keywords:** red neuronal artificial, predicción de lluvia

## Prior-free Bayes factors based on data splitting compared with geometric intrinsic Bayes factors

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Model Selections in Bayesian Statistics are primarily done with a Statistic method known as Bayes Factors. Bayes Factors require a careful assessment of prior distributions as the Intrinsic Priors

of Berger and Pericchi (1996), and a complicated integration over the parameter space. Recently researchers have been proposing alternatives to Bayes Factors which require neither integration nor specification of priors. These developments are still in a very early stage and are known as Prior-free Bayes Factors and Bayesian Stacking. This kind of method and Intrinsic Bayes Factor (IBF) both avoid specification of prior. But this Prior-free Bayes factor might need a careful choice of a training sample size. We are going to carefully study simple cases, both numerically and theoretically for a deeper understanding of their properties.

**Keywords:** prior-free, Bayes factors, cross-validation

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