SIDIM 2020 Sponsors

University of Puerto Rico at Cayey

Department of Mathematics-Physics, University of Puerto Rico at Cayey

Instituto de Investigaciones Interdisciplinarias, Universidad de Puerto Rico en Cayey

NASA Puerto Rico Space Grant Consortium

University of Iowa

East Tennessee State University

SIDIM, Inc.

Itinerario global (Global schedule)

Viernes (Friday)

Hora (Time)	NEC Amphitheater	NEC 107	NEC 108	NEC 110	NEC 111	NEC 118	NEC 120
3:00–5:00	Registration						
3:00–6:30	Exhibit Booths						
3:30–4:00	Coffee Break						
4:00–4:30		A. Rosario	J. Scott				
4:30–5:00		D. Velez	O. Tomaiconza	P. Negrón			
5:00–5:30		Y. Wang	C. Seda	R. Matos	G. Salivia		
5:30–6:00		W. Rolke	C. Serna	F. Betancourt	J. Santiago		
6:00–6:30	SIDIM 2020 Official Opening						
6:30–7:30	J. Arino						
7:30–9:00	Cocktail and Social Activity (NEC Lobby)						

Sábado (Saturday)

Hora (Time)	NEC Amphitheater	NEC 107	NEC 108	NEC 110	NEC 111	NEC 118	NEC 120
7:30–10:00	Registration						
8:00–4:00	Exhibit Booths						
7:30–9:00	Desayuno (breakfast) - NEC Lobby						
8:00–8:30							
8:30–9:00				J. Alvarez	C. Seda	F. Pabon	
9:00–9:30		L. Pericchi	J. Torres	N. De la Cruz	C. Aginsoni	T. Diaz	R. Reyes
9:30–10:00		E. Suárez	J. Velazquez	J. Jellinek	A. Ramos	R. Martinez	E. Salas
10:00–11:00	Afiches (Posters) and Coffee Break NEC (second floor)						
11:00–12:00	A. Mahabal						
12:00–1:00	F. Memoli						
1:00–2:00	Almuerzo (Lunch) - UPRC Cafeteria						
2:00–2:30		H. Mattei	A. Velez	A. Luna	E. Ramos	R. Martinez	M. Delgado
2:30–3:00		W. Rolke	J. Gonzalez		J. Vega	A. Nieves	E. Peña
3:00–3:30	Coffee Break						
3:30–4:00		R. Rivera	F. Seoanes		W. Quispe	M. Rivera	J. Ortiz
4:00–4:30		O. Disdier	R. Aparicio			W. Villafañe	F. Piñero
4:30–5:00	Sesión Administrativa (Administrative Session) (NEC Amphitheater)						

Contents

1 Itinerario Detallado (Detailed Schedule)

Horario	Lugar	Actividad
Viernes		
3:00-5:00	NEC Lobby	**Registration**
3:00-6:30	NEC Lobby	**Exhibit Booths**
3:30-4:00	NEC Lobby	**Coffee Break**
4:00-4:25		**Contributed Presentations**
	NEC 107	*Bayesian hierarchical probabilistic models to study reasonable scenarios for the population projection of Puerto Rico by 2050* Angelica M. Rosario Santos
	NEC 108	*The Hadamard product of a nonsingular general H-matrix and its inverse transpose is diagonally dominant* José Scott
4:30-4:55		**Contributed Presentations**
	NEC 107	*Adaptive α Significance Level for Linear Models* Daiver Vélez
	NEC 108	*There are no Efficient Dominating Sets in Cayley Graphs Generated by Transposition Trees of Diameter $d > 2$* Oscar Tomaiconza
	NEC 110	*A note on the Radon–Riez property* Pablo V. Negrón–Marrero
5:00-5:25		**Contributed Presentations**
	NEC 107	*An R-Package to Compute and Use Intrinsic Priors for Model Selection and Hypothesis Testing* Yekun Wang
	NEC 108	*Solvability of systems of polynomial equations with multivariate polynomials as coefficients* Carlos E. Seda Damiani
	NEC 110	*Caracterización de la sucesión de Lucas-Lehmer* Ramón Luis Matos Berrios
	NEC 111	*Una secuencia no aleatoria que aprueba las pruebas de NIST* Guarionex Salivia
5:30-5:55		**Contributed Presentations**
	NEC 107	*So, how many people did Maria kill?* Wolfgang Rolke
	NEC 108	*Value distribution of elementary symmetric polynomials and their perturbations over finite fields* Cesar Serna
	NEC 110	*Discrete Morse Theory and Rational Homology of Thompson's group T* Fernando Betancourt Velez

Horario	Lugar	Actividad
	NEC 111	*On permutation binomials of index $q^{e-1} + q^{e-2} + \cdots + 1$* Javier Santiago
6:00-6:30	NEC Amphitheatre	**SIDIM 2020 Official Opening**
6:30-7:30	NEC Amphitheatre	**Invited Presentation**: *Mathematical epidemiology in a data-rich world* Julien Arino
7:30-9:00	NEC Lobby	**Cocktail and Social Activity**
Sábado		
7:30-10:00	NEC Lobby	**Registration**
8:00-4:00	NEC Lobby	**Exhibit Booths**
7:30-9:00	NEC Lobby	**Breakfast**
8:30-8:55		**Conferencias Concurrentes**
	NEC 110	*Quantum mechanics applied to genomic sequencing* Jose R. Alvarez
	NEC 111	*The variable exponent Bernoulli differential equations* Carlos E. Seda Damiani
	NEC 118	*Creating County Health Rankings for the State of Iowa* Felix Pabon-Rodriguez
9:00-9:25		**Conferencias Concurrentes**
	NEC 107	*Alternativas de la Estadistica Bayesiana a la Crisis del p-Valor y su falta de reproducibilidad Cientifica* Luis Raúl Pericchi Guerra
	NEC 108	*Analysis and computation of the multidimensional linear complexity of periodic arrays* Jaziel Torres
	NEC 110	*Statistical mechanics of multilayer growth models: jamming and percolation* N. De la Cruz Felix
	NEC 111	*Characterization of Factorization of Certain infinite families of Polynomial of Several Variables* Carlos A. Agrinsoni Santiago
	NEC 118	*La enseñanza y aprendizaje de las funciones exponenciales y logarítmicas en estudiantes de secundaria* Tomas Diaz
	NEC 120	*Tricks in the family of Gold functions related to cryptography* Roberto Carlos Reyes Carranza
9:30-9:55		**Conferencias Concurrentes**
	NEC 107	*Mortalidad de enfermedades crónicas proyectada para el 2020-2025* Erick Suárez
	NEC 108	*On Bent Function, Boolean function properties and some interesting codes* Jose W. Velazquez Santiago

Horario	Lugar	Actividad
	NEC 110	*Inverting the ergodicity problem: adjusting statistics to dynamics (a dynamics-based solution to the problem of anharmonic densities of states* Julius Jellinek
	NEC 111	*Functions on the integers modulo n* Andrés Ramos
	NEC 118	*What does "y is defined as an implicit function of x" mean?: An application of APOS-ACE* Rafael Martínez-Planell
	NEC 120	*Binary Goppa Codes From Trace Polynomials* Eduardo Salas Ramirez
10:00-11:00	NEC (2nd floor)	**Posters and Coffee Break**
		Nanoscale multiferroic properties for memory device applications Nycole M. Arocho-Berrios
		Diffhash, a more efficient way of finding differentially expressed genes Briknie Báez Rodríguez
		Anomaly Detection in Netflows using Subspace Method Heriberto Camacho Torres
		Basic Principles of Discrete Mathematics for Error Correcting Codes Jan L. Carrasquillo López
		Computer Vision approaches to enable multi-faceted data collection of honeybees behavior in the field Jeffrey Chan
		Interventions on Chlamydia and Trichomoniasis epidemiological models based on a donovanosis eradication plan Aixamar Cosme Román
		Autoencoders vs LASSO: applications to breast cancer research Roberto Lopez Rivera
		Overview on the opinion of changing the standard p-value from 0.05 to 0.005 throughout the academic disciplines Gabriela M. Lozano Pérez
		The study of $\tau_{(n)}$-atoms Eric J. Pabón Cancel
		Automatic piano fingering data extraction from videos Carlos J. Perez Vinelli
		Epidemiological models for Chlamydia trachomatis in Puerto Rico Genesis Rivera-Bachier

Horario	Lugar	Actividad
		Codes from Cayley Graphs Doel Rivera Laboy
		Friend Factorizations Ambar C. Rodríguez-Alicea
		Modeling ultrafine particulate matter through motor vehicles emissions and its effects as a trigger of asthma Michelle N. Rosado Pérez
		Coarse-graining of Atomistic Models Using Neural Networks Bernardo A. Roque-Carrión
		Unit Groups, Quotients, and new perspectives on τ_I-factorizations Daniel A. Santiago
		on-parametric clustering diagnostic for k-means clustering solutions Eliezer Santos-León
		Mathematical Modeling of Vector-Borne Disease Dynamics with Environmental Factors Yaria Sierra-Pérez
		On the Study of f-factorizations Over Unique Factorization Domains with $f(x) = x^r$ Geraldo E. Soto-Rosa
11:00-11:50	NEC Amphitheater	**Invited Presentation**: *Machine assisted discoveries in astronomical sky surveys and possible extrapolations to biological data* Ashish Mahabal
12:00–1:00	UPRC Cafeteria	**Lunch**
1:00-1:50	NEC Amphitheater	**Invited Presentation**: *Distances between datasets: The Gromov-Wasserstein distance and applications* Facundo Memoli
2:00-2:25		**Conferencias Concurrentes**
	NEC 107	*The Projection of population decline in Puerto Rico* Hernando Mattei
	NEC 108	*Fine regularity for the elliptic anisotropic Robin problem with nonstandard growth conditions* Alejandro Vélez-Santiago
	NEC 110	*A mathematical model on the spatial spread of the Zika virus* Alibeth E. Luna Alvear
	NEC 111	*Clasificación de Microfotografías de Epidermis de Hojas utilizando Redes Neuronales Convolucionales: Análisis de la Biodiversidad con Aprendizaje Automático* Elio Ramos

Horario	Lugar	Actividad
	NEC 118	*On students understanding of implicit differentiation based on APOS Theory* Rafael Martínez-Planell
	NEC 120	*New ideas in 35 years of multivariate public key cryptography* Moises Delgado
2:30-2:55		**Conferencias Concurrentes**
	NEC 107	*Introduction to R* Wolfgang Rolke
	NEC 108	*Fundamental solutions for fractional differential equations involving fractional powers of finite difference operators* Jorge Gonzalez-Camus
	NEC 111	*Wave propagation dynamics via data-driven reduction* José Vega–Guzmán
	NEC 118	*Actitud de los estudiantes universitarios hacia las matemáticas* Aniel Nieves-González
	NEC 120	*New Trapdoor Functions Over Finite Fields of Characteristic 2 for New PKC Systems* Edgardo L. Peña Cotto
3:00-3:30	NEC Lobby	**Coffee Break**
3:30-3:55		**Conferencias Concurrentes**
	NEC 107	*Deep Learning en Ciencia de Datos* Roberto Rivera
	NEC 108	*Holomorphy of semigroup generated by the Dirichlet Fractional Laplacian* Fabian Seoanes
	NEC 111	*Temporal Outlier Detection using Dynamic Bayesian Networks and Probabilistic Association Rules* Walter Quispe-Vargas
	NEC 118	*Exámenes en pareja en una clase de iniciaciónl Cálculo Diferencial* Maribel Rivera-Rivera
	NEC 120	*New asymptotically optimal three-dimensional wavelength/space/time optical orthogonal codes for OCDMA systems* José Ortiz-Ubarri
4:00-4:25		**Conferencias Concurrentes**
	NEC 107	*La situación estadística en Puerto Rico* Orville M. Disdier
	NEC 108	*L^p-Maximal regularity for a class of degenerate fractional integro-differential equations in Banach spaces* Rafael Aparicio

Horario	Lugar	Actividad
	NEC 118	*Developing Technological Pedagogical Content Knowledge of Pre-Service Math Teachers* Wanda Villafañe Cepeda
	NEC 120	*Grassmann codes as quasi–cyclic codes and their decoding* Fernando Piñero Gonzalez
4:30-5:00	NEC Amphitheater	**Administrative Session**

2   Charlas Plenarias / Plenary Talks

Mathematical epidemiology in a data-rich world

Julien Arino, University of Manitoba

Data has become the world's most valuable resource. Most data is proprietary, but still, a lot is publicly available, for instance through open data portals. In this context, I believe that it is important that mathematical epidemiologists, when we are looking to contribute to public health policy debates, strive to use more and more "real world" information in our models. In this talk, I will discuss some personal experiences with data related issues and present simple techniques that can be used to incorporate data into models.

Machine assisted discoveries in astronomical sky surveys and possible extrapolations to biological data

Ashish Mahabal, California Institute of Technology

The Zwicky Transient Facility (ZTF) is a sky survey covering 3750 sq. degrees down to 20.5 mag from the Palomar 1.2m telescope. It has been functional for close to two years, and with its field of view of 47 square degrees the publicly released DR2 boasts of over 100 billion detections in `g//r/i` filters. The various science groups are using a variety of techniques to make interesting discoveries in this space, and there is scope to do far more. We will highlight some of the techniques and discoveries from ZTF including rare asteroids, binary systems, supernovae, tidal disruption events and so on. We will then briefly mention how similar explorations can be done using biological datasets for, say, early detection of diseases, and the possible bottlenecks.

Distances between datasets: The Gromov-Wasserstein distance and applications

Facundo Memoli, Ohio State University

Datasets can often be modelled as metric spaces in their own right. For instance point clouds inherit the metric from Euclidean space and as such two point clouds would be declared to be equivalent whenever there is an isometry mapping one into the other. In more general scenarios, one may not have a point cloud, but instead a square matrix representing distances between pairs of data points i.e. datasets are represented as finite metric spaces. Then the question becomes how to quantify the degree of similarity between two finite metric spaces. Some beautiful ideas from metric geometry due to Gromov can be applied to answer these questions. The so called Gromov-Hausdorff metric provides a way of ascribing a distance to any pair of compact metric spaces.

However, in many applications one not only has a matrix of distances for each dataset, but also a vector of weights for each point in a dataset. These objects are actually well known in geometry: Triples of the form (X,dX,muX) where (X,dX) is a compact metric space and muX is a probability measure on X are called metric measure spaces and provide a more flexible representation of data.

17

Ideas from Optimal Transport ideas then suggest a certain variant of the GH distance, called the Gromov-Wasserstein distance, which permits comparing metric measure spaces.

In this talk I will overview the construction of such distances, the optimization problems they induce, lower bounds, and applications to shape and data analysis.

3 Talleres / Workshops

3.1 Statistics and Data Science in Puerto Rico

This workshop will provide an overview of the crisis of p-values and hypothesis testing in statistics as well discussions on the frontiers and challenges of Statistics and Data Science in medicine, epidemiology and government in Puerto Rico. This workshop is sponsored by the American Statistical Association Chapter of Puerto Rico (ASA-PR).

Alternativas de la Estadistica Bayesiana a la Crisis del p-Valor y su falta de reproducibilidad Cientifica

Luis Raúl Pericchi Guerra, Centro de Bioestadística y Bioinformática y Departamento de Matemáticas, Universidad de Puerto Rico, Rio Piedras.

Por mas de 60 años desde la publicación de la Paradoja de Jeffreys y Lindley, los Bayesianos han cuestionado el test de hipótesis con valores fijos de significación Alfa, usualmente 0.05. Pero fue solo cuando los científicos aplicados demostraron que no había replicación de resultados en el 95se desató la crisis. En esta charla explicamos la razón de la falta de replicabilidad científica, así como alternativas prácticas, dentro del espíritu Bayesiano Objetivo, para lograr replicabilidad científica. Acá resumimos trabajos conjuntos con: Carlos Pereira (Brasil), María Eglee Perez (PR) y Daiver Velez (PR y Colombia).

Mortalidad de enfermedades crónicas proyectada para el 2020-2025

Erick Suárez, Departamento de Bioestadística y Epidemiología, Escuela Graduada de Salud Publica, Universidad de Puerto Rico Recinto de Ciencias Médicas.

El Departamento de Salud de Puerto Rico ha solicitada las proyecciones de la mortalidad de enfermedades crónicas de mayor frecuencia con el propósito de desarrollar el plan estratégico de salud para Puerto Rico. Para responder a esta petición se realizaron estas proyecciones de mortalidad para diferentes enfermedades crónicas utilizando modelos no-lineales con la función exponencial, logística y Gompertz. En esta presentación se presentarán los resultados más relevantes de estas proyecciones.

The Projection of population decline in Puerto Rico

Hernando Mattei, Demography Program, Graduate School of Public Health, University of Puerto Rico Medical Sciences Campus.

Puerto Ricos population has entered a phase of population decline due to a sharp drop in fertility and increasing levels of outmigration. The Census Bureau has proyected the population of Puerto Rico until the year 2100 and it shows a large decline in population size with a very old age structure. The United Nations population proyections also have Puerto Rico with a negative rate of growth until 2100. We examined the assumptions used in both proyections and found that they are likely to underestimate the rate of the decline. Expected trends in fertility and mortality make it clear that the Island will experience a long period of negative natural increase (more deaths than births) and that migration trends will determine how much the population will decline. We discuss the urgent need to develop more realistic probabilistic population projections for Puerto Rico.

Introduction to R

Wolfgang Rolke, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

This workshop will provide a short introduction to the R language. Topics to be discussed are: Why R?, setting things up, data entry with rio, basic Inferences, graphs with ggplot2, Bayesian Analysis, OpenBugs, C++ with Rcpp, and parallel and gpu programming with parallel and gpuR. Also we discuss as a case study an analysis of the UPR Admission data.

Deep Learning en Ciencia de Datos

Roberto Rivera, Universidad de Puerto Rico en Mayagüez.

En éste taller hacemos una breve introducción a los modelos que están en la vanguardia en ciencia de datos; deep learning. Hablamos de algunas variaciones y como se implementan estos tipos de modelos usando RStudio y el paquete keras.

La situación estadística en Puerto Rico

Orville M. Disdier, Director Ejecutivo Interino del Instituto de Estadísticas de Puerto Rico.

La Ley Número 209 del 28 de agosto de 2003 creó al Instituto de Estadísticas de Puerto Rico, la cual es una agencia gubernamental, bajo la rama ejecutiva, con autonomía fiscal y administrativa. La exposición de motivos en su ley orgánica revela que el Instituto fue creado debido a la apremiante necesidad de que el público cuente con información y datos para la toma de decisiones basadas en la evidencia. De igual forma, expone claramente que en ese momento en el tiempo,

existía una insatisfacción en general y múltiples críticas con la cantidad, la calidad y la prontitud de los datos producidos por las agencias gubernamentales. El Instituto comenzó a funcionar en el año 2007 y desde entonces ha hecho enormes esfuerzos para garantizarle a Puerto Rico que cuente con estadísticas completas, confiables, y de acceso rápido y universal. Hoy, trece años después, es pertinente reflexionar sobre cuál ha sido el impacto del Instituto en la situación estadística de Puerto Rico, cuáles son los avances y cuáles son los retos que aún nos quedan. Por otro lado, recientemente, la Junta de Directores del Instituto estableció un nuevo Plan Estratégico a tres años para: 1) Desarrollar una estructura administrativa que permita el funcionamiento adecuado de los procesos internos del Instituto, 2) Reestructurar el comité de coordinación estadística para que opere efectivamente y aumente el cumplimiento con la ley que rige al Instituto, y 3) Mejorar el presupuesto o capital diversificando los servicios y proyectos del Instituto para la obtención de ingresos externos. ¿Cómo se prevé que este nuevo plan aporte a un Puerto Rico con más transparencia, más datos y de mejor calidad?

4 Sesiones Temáticas/ Thematic Sessions

4.1 Mathematical Physics

Inverting the ergodicity problem: adjusting statistics to dynamics (a dynamics-based solution to the problem of anharmonic densities of states)

Julius Jellinek, Chemical Sciences and Engineering Division, Argonne National Laboratory.

Density of states (DOS) is a fundamental characteristic of systems that lies in the very foundation of statistical mechanics and all the theoretical constructs that derive from them (e.g., kinetic rate theories, phase diagrams, etc.). Knowledge of DOS is central for calculation of entropy, partition function, free energy, reaction rate constants, and other important characteristics. The accuracy of all these depends on the accuracy with which the DOS is defined. Even though virtually all real systems are anharmonic, the current practice in the computation of vibrational DOSs is largely based on the harmonic approximation. The reason is that despite major efforts over about eight decades a general and exact, yet practical in applications, solution to the problem of anharmonic DOSs stubbornly resisted resolution. The alternatives introduced are mostly limited to cases of weak anharmonicity and/or suffer from other shortcomings.

In a recent development, we formulated a general and exact solution to this long-standing problem, which is applicable to arbitrary degree of anharmonicity (i.e., any system) and that is practical and efficient in applications. Our solution is based on simulating the actual dynamical behavior of systems on the time scale of interest, short or long, as defined by the experiment and/or the nature of the process or phenomenon at hand. As a consequence, the resulting anharmonic DOSs are fully dynamically informed and, in general, time-dependent. As such, they lay the foundation for formulation of new statistical mechanical frameworks that incorporate time and reproduce exactly the actual time-averaged dynamical behavior of systems on the temporal scale of interest irrespective of whether this behavior is statistical or not in the traditional sense.

Quantum mechanics applied to genomic sequencing

Jose R. Alvarez, Escuela de Ciencias Naturales y Exactas, Pontificia Universidad Católica Madre y Maestra, Campus Santo Domingo.
Julian P. Velev, Department of Physics, University of Puerto Rico at Río Piedras.

Nano-pore sequencing via the electric current transverse to the pore is one of the most promising technique within a set of third generation technologies currently used for AND and RNA sequencing with the potential to drastically reduce costs, time and the sequencing errors inherent in second generation technologies. This technique does not require the use of chemical agents such as bisulfite treatment, labeling of bases with a fluorescent molecule, or PCR conation and amplification of ADAN samples. The method can be used to sequence nucleotide chains of very long lengths which can reduce problems with the assembly and assembly errors of the AND/RNA of current sequencing technologies.

While the technique is quite promising, high error rates should be addressed which cause a misidentification of nucleotides. A very important source of the error is the intrinsic noise in the current that arises from the dispersion of the electrons along the molecule, due to the interaction of the nucleotide that is in the pore with the neighboring nucleotides. In this work we present the mathematical modeling and computational study of the effect of structural noise on DNA/RNA sequencing by means of the electric current method through the NP. The results show that this noise exists even if the molecule is perfectly stationary with respect to the NP electrodes, and could be comparable in magnitude to extrinsic noise, introducing very large error levels. While this noise cannot be eliminated, its effects can be minimized by a judicious choice of the type of NP electrode material and the voltage applied across the electrodes. Likewise, a Bayesian-type computational sequencing algorithm was developed, which considers the correlations between current readings through neighboring bases of the DNA/RNA molecule.

Statistical mechanics of multilayer growth models: jamming and percolation

N. De la Cruz Felix, Departamento de Física, Instituto de Física Aplicada (INFAP), Universidad Nacional de San Luis, Argentina, y Departamento de Física, Instituto de Física (IFIS), Universidad Autónoma de Santo Domingo.
P.M. Centres, Departamento de Física, Instituto de Física Aplicada (INFAP), Universidad Nacional de San Luis, Argentina.
A.J. Ramirez-Pastor, Departamento de Física, Instituto de Física Aplicada (INFAP), Universidad Nacional de San Luis, Argentina.

Adsorption is a physicochemical process of great importance because of its application in the chemical industry and experimentation. In this sense, it is possible to distinguish between physical adsorption (from van der Waals forces) and chemical adsorption(characteristic of covalent bonding). In Statistical Mechanical, random sequential adsorption models (RSA) have been useful to study

physical adsorption. In that order, several theories have been formulated to explain the physical adsorption of gas molecules on a solid surface. So, RSA have been useful to study a broad class of out of equilibrium systems, particularly those related to percolation and jamming phase transitions. For both of these problems, the size and shape of the deposited objects play an important role in the kinetics of adsorption and in the final structure of the adsorbed monolayer. In the monolayer regime, the associated phenomenaoffer a basic representation of complex processes, such as adsorption of proteins on solid surfaces, particles on a biological membrane, latex spheres on a silica surface, chemisorption of large molecules, etc.. In fact, most of the studies are devoted to monolayer depositions and the development of a more realistic model, considering the formation of multilayers, has received less attention.The objective of this work is to provide a study in this direction through a model which takes into account the deposition of semi-rigid k-mers on one dimensional multilayer lattice. For this purpose, extensive numerical simulations, finite-size scaling theory and statistical analysis have been done. Then, the Jamming threshold is reported as a function of the k-mer size and the number of deposited layers. In addition, the critical exponents characterizing the jamming phase transitions have been determined. Finally, it is observed that each layer shows a virtual percolation transition.

4.2   Statistics and Data Science in Puerto Rico

Bayesian hierarchical probabilistic models to study reasonable scenarios for the population projection of Puerto Rico by 2050

Angelica M. Rosario Santos, Centro de Bioestadística y Bioinformática y Departamento de Matemáticas, Universidad de Puerto Rico, Rio Piedras.
Luis Raúl Pericchi Guerra, Centro de Bioestadística y Bioinformática y Departamento de Matemáticas, Universidad de Puerto Rico, Rio Piedras.
Hernando Mattei, Demography Program, Graduate Schoool of Public Health, University of Puerto Rico Medical Sciences Campus.

Recently, researchers have incorporated Bayesian Probabilistic Theory to include the uncertainty in the estimation of the basic demographic indicators of fertility and mortality used in the projections. In this project we considered the Bayesian Hierarchical Models for Total Fertility Rate and Life Expectancy proposed by Adrian Raftery and his colleagues to adopt the deterministic models suggested by the United Nations Population Division. We carefully choose the data to be used to justify possible scenarios for the Population of Puerto Rico and their demographic components using recent information from the World Population Prospects and the US. Census Bureau. However, since Net Migration is the most difficult demographic component to model because of its variability in the last cohort, we have been focused considering different plausible scenarios given the observed trends in migration not mainly due to climatic events. These methodologies are intended to give a scientific probabilistic framework to compare scenarios and public policies. Overall, the considered scenarios suggest a dramatic decrement in the Population of Puerto Rico for 2050 but discrepancies

among magnitudes of the population projection looks are due to estimates on Total Fertility Rate and mainly on Net Migration assumptions. According to one of the scenarios, the population of Puerto Rico will decline to 2.14 million by 2050 bounded by the 95 interval (1.8793, 2.4062). Other demographic measures are shown, like the Potential Support Ratio, to see the future implications of our results due to the age structure of the population. R packages bayesLife, bayesTfr and bayesPop were used to obtain the corresponding projections and to verify the convergence of the MCMC simulations to get the posterior distributions of the parameters.

Adaptive α Significance Level for Linear Models

Daiver Vélez, Statistical Institute and Computerized Information Systems, University of Puerto Rico at Río Piedras.
Luis Raúl Pericchi Guerra, Department of Mathematics, University of Puerto Rico at Río Piedras Campus.
María E. Pérez, Department of Mathematics, University of Puerto Rico at Río Piedras Campus.

We put forward an adaptive alpha that decreases as the information grows, for hypothesis tests in which nested linear models are compared. A less elaborate adaptation was already presented in Pérez and Pericchi (2014) for comparing general i.i.d. models. In this article we present refined versions to compare nested linear models. This calibration may be interpreted as a Bayes-non-Bayes compromise, and leads to statistical consistency, and most importantly, it is a step forward towards statistics that leads to reproducible scientific findings.

Keywords: p-value calibration; Bayes factor, linear model; likelihood ratio; adaptive alpha; PBIC

So, how many people did Maria kill?

Wolfgang Rolke, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

In the two and a half years since Hurricane Maria there has been an ongoing discussion about how many people died due to the storm. The official government estimate went from 64 to 2975. There have been six peer-reviewed studies published in scientific journals that give estimates. The first of these, "Estimating the death toll of Hurricane Maria" and published in February of 2018 in Significance magazine, was written by Dr. Rivera of UPRM and me. We also published the article "Modeling Excess Deaths After a Natural Disaster with Application to Hurricane Maria", in the journal Statistics in Medicine in October 2019. Four other articles were published in the journals "New England Journal of Medicine", "Lancet", "Journal of the American Medical Association" and "American Journal of Public Health". The now official estimate is based on the analysis in the Lancet article.

In this talk I will discuss all of these articles, their data and methodology of analysis and their results. I will discuss to what degree those results are in agreement. I will also discuss some issues that arise when a statistical analysis is of interest to the wider public, as is certainly the case here.

An R-Package to Compute and Use Intrinsic Priors for Model Selection and Hypothesis Testing

Yekun Wang, Department of Mathematics, University of Puerto Rico at Rio Piedras.
Elias Moreno, University of Granada, Spain
Luis Raúl Pericchi Guerra, Department of Mathematics, University of Puerto Rico at Rio Piedras.

Bayes Factors are better than p-values and should eventually replace them as a better scientific tool. On the other hand, Bayes Factors, except in specific instances, cannot be computed with improper flat priors as in Objective Estimation. Intrinsic Priors methodology has been the first general method to construct Objective Priors suited to Model Comparisons. Intrinsic Priors based on the Intrinsic Bayes factors proposed by J.O. Berger and L.R. Pericchi (1996) have been well known and studied. Using a small portion of samples as the training samples to train prior distributions while computing the expression via the validation set (data exclude training samples), the Intrinsic Bayes factor is an excellent approximation to Bayes factors. Through careful computations, one can attain the so-called intrinsic prior, which has been a prior computing an intrinsic Bayes factor. To celebrate 24-year-old of the Intrinsic Prior methodology and to facilitate its use, we present some ideas about our R package for computing intrinsic priors under both continuous and discrete distributions, such as normal distributions, Chi-square distributions in the continuous group, and Poisson distributions, Hardy-Weinberg equations in the discrete group. Users who input the parameters required in the system can have outputs and the plot of intrinsic priors and other relevant information. The Intrinsic Priors R-Package is planned to be available later on 2020.

5   Charlas Concurrentes / Concurrent Talks

(In alphabetical order using the last name of the speaker.)

Characterization of Factorization of Certain infinite families of Polynomial of Several Variables

Carlos A. Agrinsoni Santiago, Department of Mathematics, University of Puerto Rico at Rio Piedras.
Heeralal Janwa, Department of Mathematics, University of Puerto Rico at Rio Piedras.
Moises Delgado, Department of Mathematics–Physics, University of Puerto Rico at Cayey.

The factorization of polynomials in many variables over a field is an interesting and very complex problem. On finite fields, factorization of multivariate polynomials and irreducibly testing is very important for the construction of algebraic geometric codes. These problems are also crucial for determining whether the corresponding polynomials are permutation polynomials, or have non-linearity properties. Several authors have resolved this problem for infinite classes, and certain conjectures and open problems are formulated. We extend these results substantially involving infinite families.

In this presentation we give a characterization of the factors of a polynomial (if possible) given the polynomial satisfies certain properties. We prove the existence of absolutely irreducible factors of degree greater than one for some important classes of polynomials. We discuss applications to

coding theory and cryptography.

L^p-Maximal regularity for a class of degenerate fractional integro-differential equations in Banach spaces

Rafael Aparicio, Statistical Institute and Computerized Information Systems, University of Puerto Rico at Río Piedras.
Valentin Keyantuo, Department of Mathematics, University of Puerto Rico at Río Piedras.

Through the technique of operator-valued Fourier multipliers, we establish characterizations for well-posedness of a class of degenerate fractional integro-differential equations of order α (with $0 < \alpha < 1$). We deal with the right-sided Riemann-Liouville fractional derivative on the real axis and the Lebesgue-Bochner spaces $L^p(\mathbb{R}, X)$, $1 \leq p < \frac{1}{1-\alpha}$ where X is a given complex Banach space. When the space X has the UMD property and $1 < p < \frac{1}{1-\alpha}$, we obtain concrete conditions for well-posedness based on the concept of R-boundedness (or Rademacher boundedness) for operator families. The results are applied to some concrete equations corresponding to models relevant to the applied sciences, particularly physics, rheology, and material science.

Discrete Morse Theory and Rational Homology of Thompson's group T

Fernando Betancourt Velez, Department of Mathematics–Physics, University of Puerto Rico at Cayey.

Thompson's group T was introduced by Richard Thompson in 1965 and is an infinite simple group with a finite presentation. The homology of T was first calculated by Ghys and Sergiescu in 1987 using very complicated tools. We attempt to compute the homology of T directly from the cellular chain complex $\mathcal{C}$ associated to a natural cell complex on which T acts. To simplify calculations, we use discrete Morse theory on $\mathcal{C}$ to create a more manageable object called the Morse complex. More specifically, we construct a discrete gradient vector field on the complex $\mathcal{C}$ and use a filtration by sub-complexes to create finitely generated chain groups with which to calculate the homology.

Functions on the integers modulo n

Dylan Cruz, Department of Computer Science, University of Puerto Rico at Río Piedras.
Andrés Ramos, Department of Computer Science, University of Puerto Rico at Río Piedras.
Ivelisse Rubio, Department of Computer Science, University of Puerto Rico at Río Piedras.

Ullman and Velleman recently studied which functions from an abelian group G to itself can be expressed as a difference of two bijections from G to itself. The results have applications to juggling and bus scheduling. If $G = \mathbb{Z}_n$ one has that $a : \mathbb{Z}_n \to \mathbb{Z}_n$ can be written as a difference of bijections if and only if $\sum_{i \in \mathbb{Z}_n} a(i) \equiv 0 \pmod{n}$. We present results on a more general question: When a function from a group to itself can be expressed as a difference of two functions with the same image as a multiset. We also find connections to juggling sequences and cyclic group orthomorphisms.

New ideas in 35 years of multivariate public key cryptography

Moises Delgado, Department of Mathematics–Physics, University of Puerto Rico at Cayey.

Trapdoor functions over finite fields are the security components of multivariate cryptographic systems for public key cryptography (PKC). PKC systems use two keys, a public key and a private key. A public key used for encryption while the private key is used for decryption. It is almost impossible a decryption without knowing the private key. Public key systems are fundamental security ingredients in modern electronic communications and data storage. Current PKC systems are based on number theory principles, then could be obsolete provided the emergence of quantum computers and quantum attacks as showed by Peter Shor. In this talk we propose new ideas for designing new trapdoor functions of high degree for PKC. This high degree promises high resistance against the main attacks because of the difficulty for solving a system of multivariate polynomial equations. Current systems use low degree trapdoor functions, then they are sensitive to these attacks.

La enseñanza y aprendizaje de las funciones exponenciales y logarítmicas en estudiantes de secundaria

Tomas Diaz, Universidad Interamericana de San German.

Se presentan resultados preliminares de un estudio en progreso sobre la enseñanza y aprendizaje de las funciones exponenciales y logarítmicas en estudiantes de escuela secundaria. Como marco teórico se usa la teoría APOE (Acción, Proceso, Objeto, Esquema) y en particular la estrategia pedagógica del ciclo ACE (Actividades, discusión en clase y ejercicios) que esta teoría propone. En este estudio se utilizó el método de investigación cualitativa en un experimento de enseñanza.

Se constituyó un grupo de 16 estudiantes que estuvieron en observación. El grupo de estudiantes se dividió en dos subgrupos de 8 estudiantes cada uno. El experimento de enseñanza consistió de una prepueba, 17 actividades de aprendizaje, 6 hojas de trabajo para hacer en el hogar, entrevista semiestructurada a 9 estudiantes y posprueba. Se discutirán algunos antecedentes, el diseño del experimento, las actividades y los resultados que hasta el momento hemos obtenido. En la discusión de resultados se comenzará examinando una conjetura sobre las construcciones mentales que se esperaba fomentar con las actividades de aprendizaje. Luego de esto se reportará cómo el comportamiento observado de los estudiantes, mientras discuten las actividades; respalda o no la conjetura. Estas observaciones pueden llevar a refinar la conjetura y/o rediseñar las actividades de aprendizaje.

Keywords: crecimiento exponencial, decaimiento exponencial, logaritmos, APOE y ciclo ACE

Fundamental solutions for fractional differential equations involving fractional powers of finite difference operators

Jorge Gonzalez-Camus, Department of Mathematics, Universidad de Santiago de Chile.
Carlos Lizama, Department of Mathematics, Universidad de Santiago de Chile.
Pedro Miana, Department of Mathematics, Universidad de Zaragoza.

In this talk we study the representation of the solution (unique) for the following time/space fractional evolution equation:

$$\begin{cases} \mathbb{D}_t^\beta u(n,t) = Bu(n,t) + g(n,t), & n \in \mathbb{Z}, \ t > 0. \\ u(n,0) = \varphi(n), \ u_t(n,0) = \phi(n) & n \in \mathbb{Z}, \end{cases}$$

where we consider the operator $Bf(n) = (K * f)(n)$, with $K \in l^1(\mathbb{Z})$, $f \in l^p(\mathbb{Z})$, $p \in [1,\infty]$ and $\beta \in (1,2]$ is real number. We recall that $\mathbb{D}_t^\beta$ denotes the Caputo fractional derivative.

It is important remark the fact that fractional powers for a class of the finite difference operators have a representation through discrete convolution, involving kernels satisfying the above hypotheses, for instance the discrete fractional Laplacian and the discrete fractional left/right derivative.

Classic evolution equations (discrete in space) are involved in this work as particular case. For instance, we obtain the explicit solution for the heat equation, wave equation, Poisson equation, Airy equation, transport equation, subdiffusives and superdiffusives cases, etc.

Furthermore, we present fractional representation for certain discrete operators and its relation with the spectrum.

Keywords: Caputo fractional derivative; discrete fractional Laplacian; fundamental solutions, Wright and Mittag-Leffler functions

A mathematical model on the spatial spread of the Zika virus

Alibeth E. Luna Alvear, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.
Karen R. Rios Soto, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

Zika virus, which belongs to the Flavivirus genus, of the Flaviviridae family, is transmitted by the bite of female mosquitoes (vectors) Aedes Aegypti, as well as sexually. This disease was first identified in Macacos (Uganda, 1947) and spread in the Americas region in mid-2015. Since then, outbreaks have occurred in multiple countries and territories in South America and the Caribbean. According to the World Health Organization (WHO) and Puerto Rico Department of Health (PRDH), the first locally acquired case of Zika virus disease in Puerto Rico was identified in early December 2015 and by 2016 a total of 35,395 cases were reported on the island. In this work, a mathematical model is proposed to study the spatial dissemination of the Zika virus using a system of partial differential equations of the reaction-diffusion-advection type. The populations considered are the human and mosquito population, for which we characterize subclasses of individuals based on their epidemiological status (susceptible, infected and recovered for the human population and susceptible and infected for the mosquito population). To describe the localized spread of the Zika virus during a short period of epidemic, the human population is considered homogeneously distributed in space. The system equilibria are calculated, as well as the threshold condition R0 based on the model parameters, this condition helps determine the possible spread of Zika virus and the risk of outbreaks. The reaction-diffusion-advection model is analyzed assuming traveling wave solutions that, in turn, allow to calculate a minimum rate of virus propagation. On the other hand, with parameters for the model obtained from the literature, numerical simulations are performed to study the behavior of the traveling waves.

What does "y is defined as an implicit function of x" mean?: An application of APOS-ACE

Rafael Martínez-Planell, Departamento de Ciencias Matemáticas, Universidad de Puerto Rico en Mayagez y Universidad Interamericana San Germán.
Vahid Borji, Department of Applied Mathematics, Faculty of Mathematical Sciences, Ferdowsi University of Mashhad, Iran.

A pesar de que función implícita es un tópico básico e importante en el cálculo, es muy poca la investigación que se ha hecho de cómo los estudiantes pueden llegar a entender esta noción. En un estudio previo se documentaron dificultades comunes y falta de entendimiento de los estudiantes de este tópico y se propuso una conjetura (descomposición genética; DG) de construcciones mentales que un estudiante puede hacer para llegar a entender la noción de función implícita. En este estudio se pone a prueba y se refina la descomposición genética propuesta. El estudio comienza llevando a cabo entrevistas semiestructuradas con catorce estudiantes que recién habían terminado un curso tradicional (basado en conferencias; que sigue de cerca el libro de texto y prontuario; que no incluye actividades especiales basadas en APOE) de cálculo de funciones de una variable. En las entrevistas se exploró el conocimiento de función implícita de estos estudiantes. Luego de

esto se usó la DG para diseñar un ciclo de Actividades en computadora, discusión en el salón de Clase, y Ejercicios (ciclo ACE). El ciclo se diseñó e implantó para ayudar a que los estudiantes pudiesen hacer construcciones mentales que se habían observado causaban dificultad durante las entrevistas iniciales y estudios previos. Finalmente, se volvieron a entrevistar los mismos catorce estudiantes luego de que participaran en las actividades del ciclo ACE. Las contestaciones de los estudiantes en esta segunda entrevista muestran mejoría en su entendimiento de función implícita. Los resultados de este artículo son de importancia práctica ya que sugieren que actividades basadas en la DG propuesta tienen el potencial de mejorar en entendimiento de función implícita de los estudiantes. Además, el estudio también sugiere posibles revisiones a la descomposición genética de función implícita.

Palabras claves: cálculo, función implícita, APOE, ciclo ACE, descomposición genética

On students understanding of implicit differentiation based on APOS Theory

Rafael Martínez-Planell, Departamento de Ciencias Matemáticas, Universidad de Puerto Rico en Mayagez y Universidad Interamericana San Germán.
Vahid Borji, Department of Applied Mathematics, Faculty of Mathematical Sciences, Ferdowsi University of Mashhad, Iran.

Se aplica la Teoría APOE (Acción, Proceso, Objeto, Esquema) para estudiar el entendimiento de diferenciación implícita que tienen los estudiantes en el contexto de un curso de cálculo de funciones de una variable. Las nociones en APOE de esquema y de desarrollo de esquema en términos de la tríada Intra- (componentes del esquema esencialmente aislados), Inter- (comienzan a hacerse interconexiones), Trans- (componentes interrelacionados, se alcanza la coherencia), se usan para analizar entrevistas semiestructuradas a 25 estudiantes que recién terminaron un curso tradicional (basado en conferencias, sigue el libro de texto y prontuario muy de cerca, sin actividades especiales basadas en APOE) de cálculo de funciones de una variable. Los resultados sugieren que las nociones de regla de la cadena y función implícita juegan un rol especial en la posibilidad de alcanzar la coherencia del esquema. Para esto los estudiantes necesitan construir al menos una conceptualización de proceso de función implícita, y un esquema de regla de la cadena con coherencia dada por composición de funciones. Los estudiantes también necesitan construir relaciones entre función implícita y cada uno de tres componentes del esquema de diferenciación implícita: función explícita (para entender el significado gráfico de función implícita), derivada (para consistentemente pensar y tratar a y como una función desconocida de x durante diferenciación implícita), y reglas de diferenciación (para llevar a cabo los cómputos necesarios). El estudio muestra que estudiantes que toman un curso introductorio tradicional de cálculo puede esperarse que tengan dificultad con las ideas principales de diferenciación implícita a menos que se diseñen actividades especiales para ayudarlos a hacer las conexiones necesarias entre los componentes del esquema de diferenciación implícita.

Palabras claves: cálculo, diferenciación implícita, APOE, esquema, descomposición genética

Caracterización de la sucesión de Lucas-Lehmer

Ramón Luis Matos Berrios, Departamento de Ciencias Naturales, Matemáticas, Universidad de Puerto Rico en Carolina.

En este trabajo se relaciona la desviación estándar σ_{I_n} del subconjunto de números naturales $I_n = \{k \in \mathbb{N} : k \leq n\}$ con la sucesión (s_n) de Lucas-Lehmer, la cual se vincula con sucesiones de triángulos de Herón. De esta forma, se obtienen caracterizaciones de dicha sucesión y sus relaciones con ecuaciones de Pell. En las caracterizaciones se usan las sucesiones del perímetro, del área de triángulos de Herón y sucesiones vinculadas a las mismas.

A note on the Radon–Riez property

Pablo V. Negrón–Marrero, Department of Mathematics, University of Puerto Rico at Humacao.

The "Radon–Riez" property gives sufficient conditions for extracting strongly convergent subsequences out of weakly convergent ones in Banach spaces. The key assumption is that the norms of the elements of the weakly convergent sequence, converge to the norm of the weak limit. This result is well known for Hilbert spaces of which L_2 is a special case, but it is also true for uniformly convex Banach spaces of which L^p with $1 < p < \infty$ are special cases. In this expository talk I will present a proof of this more general result.

Keywords: weak convergence, strong convergence, uniformly convex space

Actitud de los estudiantes universitarios hacia las matemáticas

Wanda Velázquez Rosado, Instituto de Estadística y Sistemas Computadorizados de Información, Universidad de Puerto Rico en Río Piedras.
Wanda Villafañe Cepeda, Departmento de Programas y Enseñanza, Universidad de Puerto Rico en Río Piedras.
José Vega Vilca, Instituto de Estadística y Sistemas Computadorizados de Información, Universidad de Puerto Rico en Río Piedras.
Aniel Nieves-González, Instituto de Estadística y Sistemas Computadorizados de Información, Universidad de Puerto Rico en Río Piedras.

La literatura indica que uno de los factores que puede incidir en el fracaso de los estudiantes en los cursos de matemáticas a nivel universitario es la actitud que éstos tienen hacia esta disciplina. Específicamente, varios autores coinciden en señalar que existe una relación positiva entre la actitud de los estudiantes hacia la matemática y el desempeño en esta materia (Evans, 2007; Lin y Huang, 2015; Perry, 2011). El tener actitudes negativas hacia las matemáticas se traduce en un pobre compromiso con la clase, lo que inevitablemente conduce al fracaso (Mayes et al., 2008).

Precisamente el objetivo principal de este estudio es investigar la actitud hacia las matemáticas

que tienen estudiantes matriculados en los cursos Matemática Finita y Precálculo para Administración de Empresas (MECU 3035) y Métodos Cuantitativos para Administración de Empresas I (MECU 3031) que se ofrecieron en el Recinto de Río Piedras durante el año académico 2019-2020. Como parte de la metodología del estudio se llevó a cabo el proceso de traducción y adaptación del inglés al español del instrumento *Attitudes toward Mathematics Inventory* de Lim y Chapman (2013) por expertos en el área y un estudio piloto para afinar la traducción y adaptación del instrumento. Como segunda fase, se administró el instrumento y se llevó a cabo el análisis estadístico correspondiente, esto es: (1) Estadística descriptiva para cada variable; (2) pruebas de hipótesis para diferencia en medias de puntaje para cada variable y para determinar diferencias en género en cuanto la actitud hacia las matemáticas; (3) prueba de esfericidad de Barlett; (4) análisis factorial exploratorio; y (5) análisis factorial confirmatorio.

Palabras claves: precálculo, métodos cuantitativos, actitud hacia las matemáticas, género

New asymptotically optimal three-dimensional wave-length/space/time optical orthogonal codes for OCDMA systems

José Ortiz-Ubarri, Department of Computer Science, University of Puerto Rico at Río Piedras.

We present new families of three-dimensional (3-D) optical orthogonal codes for applications to optical code-division multiple access (OCDMA) networks. The families are based in the extended rational cycle used for the 2-D Moreno-Maric construction. The new families are asymptotically optimal with respect to the Johnson bound.

Creating County Health Rankings for the State of Iowa

Felix Pabon-Rodriguez, Department of Biostatistics, University of Iowa.
Roger Netherton, University of Missouri.
Jacob Oleson, Department of Biostatistics, University of Iowa.

For modern civilizations, the understanding of the overall health of the population is of crucial importance. In an effort to monitor the overall health of the people in the United States, the United States Department of Health and Human Services created the Healthy People program. Using the format of the latter, the Iowa Department of Public Health created the Healthy Iowans program. To take this healthy initiative one step further, we have seen the creation of the Healthiest State Initiative for the state of Iowa, with communities ranked according to health status. We participate in health assessment by ranking the 99 counties in Iowa from the most healthy to the least healthy according to a variety of health measures. Due to the rural nature of Iowa, many of these measures have small counts, which lead to highly variable estimates. We use Bayesian spatial smoothing techniques to arrive at reliable estimates of the chosen measures of health. We then combine those measures in a statistical model to provide health status ranks for the 99 counties in Iowa, and we display the ranking on a map of the state. To see how the rank of the counties change based on different health factors, a simple dynamic interface (Shiny application) was created.

New Trapdoor Functions Over Finite Fields of Characteristic 2 for New PKC Systems

Edgardo L. Peña Cotto, Department of Mathematics, University of Puerto Rico at Rio Piedras.
Eduardo Salas Ramirez, Department of Mathematics–Physics, University of Puerto Rico at Cayey.
Moises Delgado, Department of Mathematics–Physics, University of Puerto Rico at Cayey.

To ensure the transmission of digital information over an open communication channel is one of the crucial problems in the digital age of our modern world. There exist various kinds of cryptographic algorithms. We work specifically towards those regarding to Public Key Cryptography (PKC). These algorithms consider two types of keys, a private key for decryption and a public key for encryption. In this talk we propose a new probable PKC system using almost permutations over Finite Fields of characteristic 2. Current proposed PKC systems, such as Matsumoto-Imai system and Hidden Field Equation systems, have revealed some weakness against algebraic attacks because of the small degree of their security components. For our design we propose to use an almost permutation of high degree to ensure high resistance. Our method consists of perturbing a permutation f over a given finite field and consider the multivariable vectorial function $\overline{f}$ to construct the public key. We recover the inyectivity by considering a subspace of the field.

Grassmann codes as quasi–cyclic codes and their decoding

Fernando Piñero Gonzalez, Department of Mathematics, University of Puerto Rico at Ponce.
Prasant Singh, Department of Mathematics, University of Tromsø.

Grassmann codes are linear codes used to study several properties of the Grassmannian over finite fields. Several things are known about Grassmann codes, for example, their lengths, dimension

and minimum distance. In this work we use the automorphism group of the Grassmannian and the cyclic multiplicative group of finite fields in order to present Grassmann codes as quasi–cyclic codes. The quasi–cyclic code presentation also leads to an product code decoder.

Temporal Outlier Detection using Dynamic Bayesian Networks and Probabilistic Association Rules

Walter Quispe-Vargas, College of Business, Mathematical Science, and Computing and Information Science and Engineering (CISE), University of Puerto Rico at Mayagüez.
Edgar Acuña-Fernández, College of Business, Mathematical Science, and Computing and Information Science and Engineering (CISE), University of Puerto Rico at Mayagüez.

Temporal datasets provide records of the evolution and dependencies of random variables over time. Recently, there has been an increase in the application of temporal datasets in areas such as intrusion detection, fraud detection, activity recognition, etc. Interesting temporal outliers are anomalies that incorporate important or new information and contradict the causal probabilistic relationship in the domain knowledge described in a temporal dataset. One main objective of Data Mining is to discover interesting temporal anomalous patterns. Moreover, provide contextualization of the interestingness of the reported outliers. Most of the methods used to discover temporal outliers are reduction-based, losing valuable information in the discovery process. On the other hand, there are scarce studies about the interestingness of reported temporal outliers. Even less, to provide contextualization of the anomaly causes.

This research work deals with the problem of discovering these interesting temporal outliers in datasets. We present probabilistic association rules as measures to discover interesting temporal outliers based on domain knowledge that has been learned and represented by a Dynamic Bayesian Network. The two probabilistic association rules: i) *low support & high confidence*, and ii) *high support & low confidence*, were used to identify scenarios where the discrepancies between prior and conditional probabilities are significant. Our novel approach coalesces both methods. It allows us to discover interesting temporal outliers and provide contextualization in the form of relational subspaces, under the proposed methodology called "Domain Specific Temporal Anomalous Patterns."

The experimental results show that our approach can detect genuine temporal outliers and provide relational subspaces to explain the probable causes of the reported outliers, with promising efficiency measures. In this way, our technique becomes a state of the art method to discover interesting temporal outliers in temporal datasets. Designed to provide contextual information of the reported outliers; this, in turn, can be used to improve our understanding of the domain knowledge and the underlying temporal data generating process.

Keywords: interesting temporal outliers, dynamic Bayesian networks, probabilistic association rules

Clasificación de Microfotografías de Epidermis de Hojas utilizando Redes Neuronales Convolucionales: Análisis de la Biodiversidad con Aprendizaje Automático

Elio Ramos, Departamento de Matemática, Universidad de Puerto Rico en Humacao.
Adriana Márquez, Departamento de Biología, Universidad de Puerto Rico en Humacao.
Jamilys Rivera, Departamento de Biología, Universidad de Puerto Rico en Humacao.
Alexander Santos, Departamento de Matemática, Universidad de Puerto Rico en Humacao.
Denny S. Fernández, Departamento de Biología, Universidad de Puerto Rico en Humacao.

Presentamos los resultados de un estudio de clasificación de microfotografías de réplicas de las superficies de hojas utilizando redes neuronales y aprendizaje profundo (deep learning). Las hojas fueron colectadas de árboles y arbustos, en un bosque secundario adyacente al campus de la Universidad de Puerto Rico en Humacao. Las imágenes consisten de una muestra de 11 clases (especies de plantas) en donde se puede apreciar la epidermis del lado adaxial (superior) y del lado abaxial (inferior) de la hoja a escala microscópica (100X). Para mejorar el rendimiento de la clasificación se utilizó la técnica de aumento de datos (data augmentation), extendiendo la cantidad de muestras del conjunto de entrenamiento. Se construyó un modelo de clasificación basado en una arquitectura de red de convolución utilizando el paquete Keras/Tensorflow de Python. Resultados preliminares para las 11 clases del lado adaxial indican una precisión de clasificación de 97%.

Palabras claves: aprendizaje automático, clasificación, aprendizaje profundo, procesamiento de imágenes

Tricks in the family of Gold functions related to cryptography

Roberto Carlos Reyes Carranza, Department of Mathematical, University of Puerto Rico at Río Piedras.

Vectorial Boolean functions are useful in private key cryptography for designing block ciphers. Current research focuses mostly on building APN or differential 4 uniform functions. The Gold functions is the APN family mostly studied. In this work we show cases of functions of the Gold family with a considerably bad differentiability, which is relatively scarce in the literature. On the other hand, we also show the existence of functions of the Gold family whose Extended Walsh - Hadamard Spectrum is not of the form 2^n, $2^{\frac{n+2}{2}}$, $2^{\frac{n}{2}}$, 0 for n even, and is not of the form $2^{\frac{n+1}{2}}$, 0 for n odd.

Keywords: vectorial boolean functions, Walsh spectrum, APN, differential 4 uniform

Exámenes en pareja en una clase de iniciaciónal Cálculo Diferencial

Maribel Rivera-Rivera, Departamento de Ciencias Naturales, Universidad de Puerto Rico en Carolina.

En esta presentación discutiré los hallazgos de mi investigación cualitativa en educación matemática sobre el aprendizaje del cálculo en estudiantes a nivel universitario. La investigación fue realizada analizando el discurso matemático de los estudiantes durante sus trabajos en pareja al contestar los exámenes del curso. El marco teórico de la investigación estuvo basado en la teoría socio-cultural y el Marco Comunicacional (Commognitive framework) de la investigadora Anna Sfard. Existen algunas investigaciones a nivel universitario realizadas analizando el discurso matemático de los docentes. Sobre el aprendizaje hay pocas investigaciones y estas abordan el discurso escrito de las demostraciones de subgrupo en álgebra abstracta y la exploración sobre el concepto de derivada. Mi investigación analiza el discurso de los estudiantes durante el proceso de aprendizaje al contestar exámenes en pareja sobre los temas de límite, continuidad, derivada y diferenciabilidad. El diseño fue una investigación en la acción (Kemmis y McTaggart, 2005), en el escenario natural del salón de clases con tres parejas de estudiantes del curso de cálculo a nivel universitario. Estas parejas fueron grabadas en audio y video en seis ocasiones, cada una en periodos de reunión de dos horas aproximadamente. Los resultados confirmaron que la práctica de los exámenes en pareja provoca el discurso matemático de los estudiantes. Además, que los procesos comunicacionales que ocurrieron contribuyen al aprendizaje matemático conceptuado en la modificación e incremento de la complejidad del discurso matemático de los estudiantes. Los exámenes en pareja permitieron que mis estudiantes confirmaran su aprendizaje al comunicarse matemáticamente entre ellos. La práctica de ofrecer exámenes en pareja contribuyó al proceso de aprendizaje en varios aspectos: las interacciones sociales promovidas por el examen, el compromiso de los estudiantes con la tarea común de contestar el examen, las acciones discursivas de los estudiantes como formas de comunicación exclusivas de la actividad matemática.

Palabras claves: exámenes en pareja, marco comunicacional, aprendizaje matemático

Binary Goppa Codes From Trace Polynomials

Eduardo Salas Ramirez, Department of Mathematics–Physics, University of Puerto Rico at Cayey.
Dylan Cruz, Department of Computer Science, University of Puerto Rico at Río Piedras.
Fernando Piñero Gonzalez, Department of Mathematics, University of Puerto Rico at Ponce.

Linear Codes are algebraic objects used in digital communications. Linear codes introduce a special kind of redundancy to make communications reliable and resistant to errors. Several codes are constructed with powerful , yet simple, algebraic techniques. Binary Goppa codes are linear codes from univariate polynomial evaluation. They have promising applications in Forward Error Correction and Cryprography. In this talk we study a class of Binary Goppa codes where the defining polynomial is a trace function over an extension of degree three. We show some improvements over known bounds on the dimension of the codes.

Una secuencia no aleatoria que aprueba las pruebas de NIST

David Allen, Department of Mathematics, Borough of Manhattan Community College, City University of New York.
José La Luz, Departamento de Matemáticas, Universidad de Puerto Rico en Bayamón.
Guarionex Salivia, Computer and Information Science Department, Minnesota State University.
Jonathan Hardwick, Computer and Information Science Department, Minnesota State University.

El *National Institute of Standards and Technology* (NIST) desarrolló una serie de pruebas que determinan cuán aleatoria es una secuencia de bits generada por un algoritmo. Anteriormente presentamos un algoritmo para generar secuencias aleatorias de bits que pueden ser utilizadas en aplicaciones de encripción de acuerdo con las pruebas de NIST. En esta ocasión, exhibimos una clase de objetos de tipo combinatorial que produce una secuencia de bits apta para aplicaciones criptográficas según las pruebas de NIST. Sin embargo, esta secuencia de bits no se genera aleatoriamente. De hecho, esta exhibe rasgos altamente simétricos. Esto hace que estas secuencias sean inadecuadas para aplicaciones criptográficas a pesar de los resultados obtenidos por NIST. De esta manera, demostramos que las pruebas de NIST no identifican correctamente el nivel aleatorio de la secuencia presentada.

On permutation binomials of index $q^{e-1} + q^{e-2} + \cdots + 1$

Javier Santiago, Department of Computer Science, University of Puerto Rico at Río Piedras.
Ariane Masuda, Department of Mathematics, New York City College of Technology, CUNY.
Ivelisse Rubio, Department of Computer Science, University of Puerto Rico at Río Piedras.

The permutation binomial $f(x) = x^r(x^{q-1} + A)$ was studied by K. Li, L. Qu, and X. Chen over $\mathbb{F}_{q^2}$. They found that for $1 \leq r \leq q + 1$, $f(x)$ is a permutation binomial if and only if $r = 1$. Over the finite field $\mathbb{F}_{q^3}$ of odd characteristic, X. Liu obtained an analogous result, in which for $1 \leq r \leq q^2 + q + 1$, $f(x)$ permutes $\mathbb{F}_{q^3}$ if and only if $r = 1$. In this investigation, we complete the characterization for $f(x)$ over both $\mathbb{F}_{q^2}$ and $\mathbb{F}_{q^3}$, as well as obtain a complete characterization over $\mathbb{F}_{q^4}$. Furthermore, for $e \geq 5$, although a full characterization for $f(x)$ has yet to be obtained, we present some partial results which narrow down considerably the search for $r's$ that do indeed yield permutation binomials of the form $f(x) = x^r(x^{q-1} + A)$ over $\mathbb{F}_{q^e}$.

The Hadamard product of a nonsingular general H-matrix and its inverse transpose is diagonally dominant

Rafael Bru, Institut de Matemática Multidisciplinar, Universitat Politécnica de Valencia.
Isabel Giménez, Institut de Matemática Multidisciplinar, Universitat Politécnica de Valencia.
María Gassó, Institut de Matemática Multidisciplinar, Universitat Politécnica de Valencia.
<u>*José Scott,*</u> Instituto Tecnológico de Santo Domingo, Universidad Autónoma de Santo Domingo.

We study the combined matrix of a nonsingular H-matrix. These matrices can belong to two different H-matrices classes: the most common, invertible class, and one particular class named mixed class. Different results regarding diagonal dominance of the inverse matrix and the combined matrix of a nonsingular H-matrix belonging to the referred classes are obtained. We conclude that the combined matrix of a nonsingular H-matrix is always diagonally dominant and then it is an H-matrix. In particular, the combined matrix in the invertible class remains in the same class.

The Hadamard product of a nonsingular general H-matrix and its inverse transpose, that is, the combined matrix, has been studied previously. A complete study of the combined matrix showing its linear application can be seen in Johnson and Shapiro (1986). In the last decade, new properties of the combined matrix have been presented in Fiedler, et. al. (2010), (2011). It is in these last references where the name of combined matrix appears for the first time. It is well known that the sum by row or by column of the entries of the combined matrix of a nonsingular matrix A, (A) is exactly equal to 1. Then, if $(A) \geq 0$, the combined matrix is doubly stochastic. Bru, Gasso, and Gim (2014)-(2015) studied conditions under which the combined matrix of some classes of matrices is nonnegative. In particular, the authors have studied the nonnegativity of the combined matrix of totally positive (nonnegative) and totally negative (nonpositive) matrices and of sign regular matrices.

The combined matrix has different applications. In a process control problem, if A represents the relation among inputs and outputs, the combined matrix of A represents the relative gain array of the process. This interpretation was given in Bristol (1966) and was applied in chemistry, for instance, in McAvoy (1983). In mathematics, the combined matrix of A is used in Horn and Johnson (1991) to compute the eigenvalues of A

Results involving the Hadamard product of H-matrices can be found in Fiedler and Ptak (1967). The result where the combined matrix of a nonsingular M-matrix is also a nonsingular M-matrix was obtained by Fiedler (1962). Similar results have been obtained elsewhere. In this work we extend this result to nonsingular H-matrices. Firstly, we recall nonsingular H-matrices properties and their relations with diagonal dominance. It is proven that the combined matrix of an H-matrix of the invertible class is also an H-matrix of this class. Moreover, we obtain properties on diagonal dominance of the inverse matrix and the combined matrix of a nonsingular H-matrix belonging to the mixed class. So, we conclude that the combined matrix of a nonsingular H-matrix of the mixed class is also an H-matrix.

The variable exponent Bernoulli differential equations

Carlos E. Seda Damiani, Department of Mathematics, University of Puerto Rico at Río Piedras.
Alejandro Vélez-Santiago, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

We study a Bernoulli type first-order ordinary equation with variable exponent, namely $\frac{dy}{dx} + a(x)y = b(x)y^{p(x)}$. The functions $a(x)$, $b(x)$ are continuous and $p(x)$ is of class C^1 in a bounded interval $[\alpha, \beta]$, with $p(x) \neq 1$. Using substitution methods, we show the existence of an implicit solution to the Bernoulli problem. Numerical simulations applied to several examples are also provided.

Keywords: variable exponent differential equations, Bernoulli differential equations

Solvability of systems of polynomial equations with multivariate polynomials as coefficients

Carlos E. Seda Damiani, Department of Mathematics, University of Puerto Rico at Río Piedras.
Ivelisse Rubio, Department of Computer Science, University of Puerto Rico at Río Piedras.

In Castro, Moreno and Rubio (2020) they generalize the results of Moreno-Moreno's theorem that gives a bound for the power of a prime p to divide the number of common zeros of the multivariate polynomials $F_1, ..., F_t$. This generalization regarded the coefficients of the polynomials to be univariate polynomials over a finite field instead of plain elements of the finite field. The result led to improve a theorem of Carlitz, for the estimation of the number of variables needed so that a system of polynomial equations with coefficients in $F_q[X]$ can have non-trivial zeros. We generalize the results of Castro, Moreno and Rubio to polynomials whose coefficients are multivariate polynomials over finite fields.

Keywords: solvability, finite Fields, polynomials

Holomorphy of semigroup generated by the Dirichlet Fractional Laplacian

Valentin Keyantuo, Department of Mathematics, University of Puerto Rico at Río Piedras.
Fabian Seoanes, Department of Mathematics, University of Puerto Rico at Río Piedras.
Mahamadi Warma, Department of Mathematics, University of Puerto Rico at Río Piedras.

We study the semigroup generated by the fractional Laplacian with the zero exterior Dirichlet condition on $L^2(\Omega)$, where $\Omega \subset \mathbf{R}^N$ is an open set. We show that the semigroup generated is a strongly continuous submarkovian semigroup on $L^2(\Omega)$ and holomorphic semigroup on $L^p(\Omega)$ for every $1 \leq p < \infty$.

Value distribution of elementary symmetric polynomials and their perturbations over finite fields

Luis A. Medina, Department of Mathematics, University of Puerto Rico at Río Piedras.
Leonid Brehsner Sepúlveda, Department of Mathematics, University of Puerto Rico at Río Piedras.
Cesar Serna, Department of Mathematics, University of Puerto Rico at Río Piedras.

In this talk we establish the asymptotic behavior of generating functions related to the exponential sum over finite fields of elementary symmetric functions and their perturbations. This asymptotic behavior allows us to calculate the probability generating function of the probability that the elementary symmetric polynomial of degree k and its perturbations returns $\beta \in \mathbb{F}_q$ where $\mathbb{F}_q$ represents the field of q elements. Our study extends many of the results known for perturbations over the binary field to any finite field. In particular, we establish when a particular perturbation is asymptotically balanced over a prime field and provide a construction to find such perturbations over any finite field.

Analysis and computation of the multidimensional linear complexity of periodic arrays

Luis Quiñones, Department of Mathematics, University of Puerto Rico at Río Piedras.
Jaziel Torres, Department of Mathematics, University of Puerto Rico at Río Piedras.
Rafael Arce Nazario, Department of Computer Science, University of Puerto Rico at Río Piedras.
José Ortiz-Ubarri, Department of Computer Science, University of Puerto Rico at Río Piedras.
Ivelisse Rubio, Department of Computer Science, University of Puerto Rico at Río Piedras.

Sequences and multidimensional periodic arrays with entries in finite fields have important applications in coding theory and cryptography. The linear complexity of the sequences and multidimensional arrays are important parameters for many applications, especially those related to information security, and hardware implementation. We give a proof for the exact value of the complexity of certain arrays constructed using the composition method. We also compute the multidimensional linear complexity of families of arrays constructed by Moreno and Tirkel.

There are no Efficient Dominating Sets in Cayley Graphs Generated by Transposition Trees of Diameter $d > 2$

Italo. J Dejter, Department of Mathematics, University of Puerto Rico at Rio Piedras.
Oscar Tomaiconza, Institute of Statistics and Computerized Information Systems, University of Puerto Rico at Rio Piedras.

In this talk we discuss the existence or non-existence of perfect dominating sets in Cayley graphs generated by transposition trees. Our study allows us to identify when a Cayley graph has a perfect dominating set. A previous work proved the existence of efficient dominating sets in Cayley graphs generated by transposition trees of diameter 2. A subsequent paper determined the non-existence of efficient dominating sets generated by trees of diameter 3. In the present endeavor, we establish the non-existence of perfect dominating sets in Cayley graphs of diameter larger than 2, thus extending and completing the study generated by the previous results.

Keywords: Cayley graph, efficient dominating set, sphere packing

Wave propagation dynamics via data-driven reduction

Aaron Phillips, Department of Mathematics, Lamar University.
José Vega–Guzmán, Department of Mathematics, Lamar University.

Mathematical methods driven by data are increasingly important for characterizing complex systems in different fields of the natural sciences and beyond. Lately data-driven approaches have been essential tools for uncovering processes that are nearly impossible to handle with the available analytical tools. Such data-driven methods aim to exploit a relatively small subset of the full space where low-dimensional models can be used to describe the evolution of the full system. Historically, those solutions can often be approximated through dimensionality reduction methods such as Principal Component Analysis and Singular Value Decomposition. More recently, dynamic mode decomposition (DMD), and its Koopman generalization, have been used to provide a simple representation based on low-rank, spatio-temporal coherent structures. The approximation of the Koopman operator via DMD is critically important for the evaluation of the operator from data. In this work we consider an application of the Koopman theory to a nonlinear partial differential equation arising in the field of optical metamaterials. We demonstrate that the observables carefully chosen for constructing the Koopman operator are decisive for constructing a precise approximation of the nonlinear dynamics of wave propagation. It is found that its effectiveness depends sensitively on the careful choice of observables.

Keywords: data-driven methods; singular value decomposition; dynamic mode decomposition, Koopman operator, wave propagation

On Bent Function, Boolean function properties and some interesting codes

Jose W. Velazquez Santiago, Department of Mathematics, University of Puerto Rico at Rio Piedras.
Heeralal Janwa, Department of Mathematics, University of Puerto Rico at Rio Piedras.

Error free information transfer is a problem faced in society. Information travels trough channels that contain noise such that errors are introduced in the message. Error correcting codes add redundancy to the information and as such allow the receiver to obtain the intended message. Error correcting codes of length n, dimension m and minimum distance d can be written as [n,m,d] codes. There are multiple families of error correcting codes, such as the Reed-Muller and Hamming Codes. Hamming codes are a family of one error correcting codes with the property that their parity check matrix (an k x n matrix, k = n-m) can be written as decreasing powers of the primitive element in $GF(2^k)$. It has been showed by Janwa and Wilson that you can construct a 2 error correcting code from these one error correcting code by applying the Gold and Kasami-Welch non-linear functions $(f(x) = x^{2^L+1}, f(x) = x^{2^{2L}-2^L+1}$ when (k,L) = 1. However, this is the same condition needed for the Gold and Kasami Near-bent/bent functions to exist, showing a connection between good code construction and bent functions.

   In this work, we exploit the properties of Boolean functions and binary sequences on bent functions and construct codes based on these. We also build an algorithm to identify bent functions in 2 and 4 variables, near-bent functions in 3 and 5 variables and their properties. Some of the properties studied include non-linearity, auto correlation, symmetry and the global avalanche criterion (a cryptographic property of Boolean functions). We show that most bent functions share the same properties and also compare the computational results to the theoretical ones.

Fine regularity for the elliptic anisotropic Robin problem with nonstandard growth conditions

Maria-Magdalena Boureanu, Department of Applied Mathematics, University of Craiova, Romania.
Alejandro Vélez-Santiago, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

We develop the theory of anisotropic Sobolev spaces over general classes of domains, and consider the realization of the elliptic anisotropic Robin boundary value problem of nonstandard growth structure over such domains. Then we establish the solvability and global regularity results for this Robin problem. The results are obtained under minimal assumptions on the domain, and on the

interior and boundary data.

Developing Technological Pedagogical Content Knowledge of Pre-Service Math Teachers

Omar Hernández Rodríguez, Universidad de Puerto Rico en Río Piedras.
Wanda Villafañe Cepeda, Universidad de Puerto Rico en Río Piedras.
Gloriana González, Universidad de Illinois.

Describiremos el proyecto interinstitucional "Developing Technological Pedagogical Content Knowledge of Pre-Service Math Teachers" que tiene como meta el establecimiento de prácticas para ayudar a los futuros maestros de matemáticas del nivel secundario a diseñar lecciones incorporando tecnología que permite la interconectividad en el salón de clases. El objetivo principal es desarrollar el conocimiento del contenido pedagógico tecnológico (TPACK, por sus iniciales en inglés) para mejorar las capacidades de los estudiantes en cuanto a la compresión matemática, la fluidez en los procesos, la capacidad estratégica, el razonamiento adaptativo y la disposición productiva. Se propone la reingeniería del curso de métodos EDPE 4030 de tal manera que se integran las estrategias de Lesson Study, el TPACK, la orquestación instrumental y los espacios híbridos.

Se espera que la colaboración entre los diferentes miembros del equipo genere modelos que mejoren la transición de los cursos de métodos a los de prácticas clínicas. Las preguntas de investigación que explorará el proyecto incluyen las siguientes. ¿De qué manera los futuros maestros aprenden a aplicar los principios de planificación de orquestaciones instrumentales e implementar lecciones de matemáticas para promover la competencia matemática de los estudiantes? ¿Qué apoyos de los maestros cooperadores permiten a los futuros maestros a aplicar la orquestación instrumental en sus lecciones?, ¿cómo el modelo modificado de Lesson Study brinda oportunidades para crear espacios híbridos que conectan los cursos de métodos y las experiencias clínicas? A través del análisis de videos, diarios de estudiantes y entrevistas, el proyecto construirá un caso basado en evidencia que examina la integración de los cursos de métodos y las experiencias clínicas.

6   Afiches / Posters

(In alphabetical order using the last name of the <u>presenter</u>.)

Nanoscale multiferroic properties for memory device applications

Nycole M. Arocho-Berrios, et. al., Department of Mathematics and Physics, University of Puerto Rico at Cayey.

Multiferroic materials show magnetically driven ferroelectricity. They have fascinating properties that can be used in novel technological applications such as fast-writing, power-saving, and nondestructive data storage. However, because multiferroicity is typically observed at low temperatures, it is highly desirable to develop multiferroic materials that can be observed at room temperature. Furthermore, miniaturizing multiferroic is not an easy task. Searching for robust multiferroic properties at the nanoscale has therefore been a recurrent challenge. Due to the evolution of device miniaturization and the potential applications for new nonvolatile memory devices, the ferroelectric/multiferroic tunnel junctions are candidates for the integration of robust ferroelectricity at the nanoscale into next-generation memory and logic devices. A way to exploit these properties are to use them in multiferroic tunnel junctions (MFTJs). $Pb(Zr_{0.53}Ti_{0.47})_{0.60}(Fe_{0.5}Ta_{0.5})_{0.40}O_3$ (PTZFT) is a single-phase multiferroic material. In order to study thickness effect on electrical and magnetic properties from thicker to ultrathin films, we have grown films with thicknesses from 7 to 80 nm of PZTFT on $LSMO/(LaAlO_3)_0.3(Sr_2AlTaO_6)_{0.7}$ (LSMO/LSAT) (001) substrate deposited by pulsed laser deposition technique. Well saturated ferroelectric loops were observed for PZTFT films with a remanent polarization of 32, 25 and 10 mC/cm^2 for films with thicknesses of 80, 50 and 20 nm respectively. An enhanced saturated magnetization (Ms) was observed with increased of PZTFT layer thickness in PZTFT/LSMO structures. The average Ms values for PZTFT/LSMO heterostructures were 33, 25, and 15 emu/cm3 for 80, 50, and 20 nm respectively, at 300 K. Enhancement in magnetization with increase in PZTFT thickness may be due to the interface effect between PZTFT/LSMO layers. Piezo force microscopy measurements for 7 nm ultrathin PZTFT films showed a clear and reversible out-of-plane phase contrast above 3 V, which indicates the ferroelectric character of ultra-thin films. Magnetic force microscopy shows magnetic stripe domain in ultrathin films. This multiferroic properties at nanoscale level give the possibility to make multiferroic tunnel junction devices.

Diffhash, a more efficient way of finding differentially expressed genes

Briknie Báez Rodríguez, Department of Computer Science, University of Puerto Rico at Río Piedras.
Luis F. Mieses Gómez, Department of Computer Science, University of Puerto Rico at Río Piedras.
Humberto G. Ortiz-Zuazaga, Department of Computer Science, University of Puerto Rico at Río Piedras.

Bioinformatics is a very challenging field as interpreting biological data is not an easy task, especially when you have datasets that are filled with irrelevant data. In order to deal with that, people in the bioinformatics field prefer to look only for differentially expressed genes as usually they are more meaningful when it comes to finding anomalies. But there is another issue, because usually if you would like to have a dataset of differentially expressed genes you will need to use the following pathway. First take your RNASeq, assemble the reads, align those reads to the assembled transcriptome and then filter out the non-differentially expressed genes which will require a lot of time and computational power. The solution we offer is to find differentially expressed genes before assembling the data by using k-mers, hashing and other probabilistic approaches. This method will reduce assembly time since you will only need to assemble the most significant genes instead of the entire dataset as the standard method states.

To test our method we are going to use a dataset from the sea cucumber, the same dataset used by Mashanov and Garcia-Arraras in the paper Transcriptomic changes during regeneration of the central nervous system in an echinoderm. Mashanov and Garcia-Arraras use a standard procedure to find differentially expressed genes and we will compare their results to the results of our method.

Anomaly Detection in Netflows using Subspace Method

Heriberto Camacho Torres, Department of Computer Science, University of Puerto Rico at Río Piedras.
Amarilys Rodriguez Galarza, Department of Computer Science, University of Puerto Rico at Río Piedras.
Humberto G. Ortiz-Zuazaga, Department of Computer Science, University of Puerto Rico at Río Piedras.

The detection of anomalies in an entire network is an incredibly daunting task, even with todays advancements in technologies. To that end, this research experience was aimed at learning techniques used for scanning and processing Network Flow data. By use of SiLK and Python, information was gathered that revealed how strong a presence each source IP address has in the network. This result will serve as the foundation for the use of the Subspace method, which works by differentiating anomalous behaviour within the network. This new data obtained will allow the identification of what type of attack is being used, and possibly identify a point of origin. Understanding and

refining this method is what will allow us to generate a training dataset for our future work, the use of machine learning algorithms that can automate the detection process, with accurate results.

Basic Principles of Discrete Mathematics for Error Correcting Codes

Jan L. Carrasquillo López, Department of Mathematics-Physics, University of Puerto Rico at Cayey.
Melanie S. De Jesús Gómez, Department of Mathematics-Physics, University of Puerto Rico at Cayey.
Moises Delgado, Department of Mathematics-Physics, University of Puerto Rico at Cayey.

For a long time ago, data transmission has become a usual operation on daily interactions such as texts, emails, purchases, signatures, drawings, etc. For this transmission of data to occur, it must go through a channel such as orally, by touch, sound, internet, etc. However, public channels are not always as effective, since there's the possibility of interference (noise) that can cause the data to be erased, or altered, and be received incorrectly. Moreover, this can be a big problem when the transmitted regular or sensitive data is classified. To avoid this issue, it came to mind of mathematicians to create codes with the capacity of detecting and correcting errors so the data could be received as expected. Binary Hamming codes are classical linear 1-error correcting codes invented by R. Hamming around 1950, which are implemented in several ways. These codes are defined by a binary matrix, a generator matrix or a parity check matrix. In this work, based on Hamming codes ideas, we analyze error-correcting codes by using basic tools of mathematics, in particular, basic combinatorics. Our objective is to find simple methods for constructing 1-error correcting codes, and then extend these ideas to construct multiple error-correcting codes. Finally, we also hope to study encoding and decoding algorithms for these codes.

Computer Vision approaches to enable multi-faceted data collection of honeybees behavior in the field

Jeffrey Chan, et. al., Department of Mathematics, University of Puerto Rico, Río Piedras.

Bees are critical for our sustainability, particularly as they play a crucial role in agriculture as natural pollinators. Meanwhile, recent studies have shown that phenomena such as colony collapse disorder are causing the diminution of colonies. The specific factors are not yet fully understood due to the interaction of multiple possible causes such as pesticides, contamination, and diseases. Therefore diversity of data is required for the analysis of bees in their environment in order to identify problems and preserve colony health.

In this poster, we present a collection of computer vision techniques that together provide powerful tools to enable the collection of multiple facets of honeybee behavior in their colony in the field using video cameras. Two main aspects are considered 1) foraging behavior as observed at the colony entrance, 2) content of the hive frames including brood and storage of resources such as honey and pollen. The foraging bees facet consist of a) bee detection and tracking based on the pose estimation in the entrance of the hive, b) identification based on the appearance and, c) event detection (carrying pollen, fanning). Convolutional Neural Networks (CNNs) have state of the art performance on tasks such as image classification, detection and segmentation, and are used in multiple parts of the analysis. We based our bee identification on CNNs and supervised learning to estimate the part keypoints and skeleton of the bees. In identification we used CNNs for feature extraction to encode the identity in a low dimensional space based on the extracted features. Due to the lack of label data for the identity we trained CNNs in an unsupervised manner to learn features to discriminate between different identity bees. For hive frame analysis, CNNs were used to classify images of the individual cells based on their content (brood, capped brood, pollen, etc). Transfer learning was used to speed up the training process for CNNs. Due to limitations on the network and high volume of data to be processed, the transmission of the videos to a server for analysis may not be practical when deployed in the field. For this reason, we will also present how processing can be performed on-site, using a low-power edge computing system with GPU acceleration and the challenges this brings to adapt to existing approaches. This system could collect data in multiple colonies in real-time to study the behavior of bees to identify behavior patterns, associated genes expression with behavior, population modeling and discover potential biomarkers of the environment.

Interventions on Chlamydia and Trichomoniasis epidemiological models based on a donovanosis eradication plan

Frances M. Aponte Caraballo, Department of Biology, University of Puerto Rico at Cayey.
Aixamar Cosme Román, Natural Sciences Department, University of Puerto Rico at Cayey.
Mayteé Cruz Aponte, Department of Mathematics and Physics, University of Puerto Rico at Cayey.

Sexually transmitted diseases (STDs) are an epidemic driven by human behavior, such is the case of Chlamydia and Trichomoniasis within the Puerto Rican heterosexual population between 15-35 years old. We aimed to apply an isolation-type quarantine intervention to Chlamydia and Trichomoniasis to study the possibility of eradication or containment of these epidemics. The goal is to asses if the measurements that the Australian Government took to eradicate the Donovanosis epidemic and control HIV cases could be applied to these STDs. We will study the epidemic development by calculating the basic reproductive number (R0) of the mathematical models of Chlamydia and Trichomoniasis with and without intervention. Then, assess if these interventions in fact can eradicate or only lessen the epidemic. Our preliminary results show that, in effect, the R0 of Chlamydia and Trichomoniasis decreases significantly, but quarantine was not enough to eradicate the diseases. The reason was partially due to the asymptomatic individuals that are not yet diagnosed and continue spreading the infections. For Chlamydia de R0 decreases 57.28% for women and 52.73% for men and for Trichomoniasis the R0 decreases a 14.04% if this intervention is implemented. However, we want to combine additional interventions to figure a plan to eradicate these STDs epidemics in Puerto Rico.

Autoencoders vs LASSO: applications to breast cancer research

Roberto Lopez Rivera, Department of Computer Science, University of Puerto Rico at Río Piedras.
Miguel Hernandez Betancourt, Department of Computer Science, University of Puerto Rico at Río Piedras.
Juan Hernandez-José, Department of Computer Science, University of Puerto Rico at Río Piedras.
Humberto G. Ortiz-Zuazaga, Department of Computer Science, University of Puerto Rico at Río Piedras.

Breast invasive cancer (BIC) is estimated to affect 276,000 people in the U.S. this year. The average 5-year survival rate for women with BIC is around 90In this work we use a cancer-related database from The Cancer Genome Atlas (TCGA). To analyze this data denoising autoencoders (DA) were used to identify patterns in the genomic data of the patients. Since the standard learning algorithms, which are used to analyze similar types of data, are either dependent on what is already

known in the field of biology or unable to detect the most significant signals in the data. The DA method works by integrating "noise" into the training data, the autoencoder reconstructs it into the original uncorrupted data thus making it better at identifying and discerning those significant signals. The DA is able to discern the breast cancer tumors from normal samples and it also has a highly predictive feature of patient survival that is more predictive than the commonly used markers such as tumor grade and ER status. We will compare results from the denoising autoencoder with a regularized linear model (LASSO) trained to discern if there is a gene related to patients survival rate to the BIC treatments and see if the results surpass those of the Lasso model.

Keywords: machine learning, gene expression

Overview on the opinion of changing the standard p-value from 0.05 to 0.005 throughout the academic disciplines

Erick M. Ivanovich Méndez, Department of Mathematics, University of Puerto Rico at Río Piedras.
Gabriela M. Lozano Pérez, Department of Mathematics, University of Puerto Rico at Río Piedras.
Luis Raúl Pericchi Guerra, Department of Mathematics, University of Puerto Rico at Río Piedras.

The p-value provides a threshold for defining statistical significance. Over the years, there has been a lack of reproducibility on studies that the statistician community believes have to do with the threshold being too high (i.e., P ¡ 0.05). We study the question of how people from different scientific research communities feel about this problem. We looked at the article "Redefine statistical significance by DJ Benjamin where they propose to change it from P¡0.05 to P¡0.005 in order to reduce the increasing rate of false positives in new discovery claims, and increase the Bayes factor of an alternative hypothesis, making evidence "substantial" or "strong" from the current 'weak', according to conventional Bayes factor. We utilized the articles that cited this paper in order to conduct our research. Utilizing keywords and Python programming language, we are able to categorize and organize the citations more efficiently. We plan to use this data in order to come to conclusions about why that is.

Keywords: significance testing, statistical significance, replicability, p-value

The study of $\tau_{(n)}$-atoms

Eric J. Pabón Cancel, Department of Physics, University of Puerto Rico at Mayagüez.
Reyes M. Ortiz-Albino, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

The theory of $\tau_{(n)}$-factorizations was first defined in 2006 by Anderson and Frazier; since 2007, Hamon, Ortiz, Juett, among others have developed this theory. Many results have been obtained in terms of algebraic structure classification, but very little about the types of elements that have arisen. For example, is the concept of $\tau_{(n)}$-prime elements. Unfortunately the only prop-

erties we know is a characterization of their form (that is, $p_0^{\epsilon_0} \cdot p_1^{\epsilon_1} \cdots p_k^{\epsilon_k}$ is a $\tau_{(n)}$-prime where $= n = p_1^{n_1} \cdot p_2^{n_2} \cdots p_k^{n_k}$, each n_i is a non-negative integer, $\epsilon_i \in \{0, 1\}$ and the p_i's are non-associated primes); and the $\tau_{(n)}$-factorization into $\tau_{(n)}$-primes are unique, if they exist.

This project has the focus to try to characterize the properties and distribution of $\tau_{(n)}$-primes. Therefore, we have developed a list of the first $152,819$ $\tau_{(2)}$-primes and $136,419$ $\tau_{(3)}$-primes using an algorithm from the internet that gives us any quantity of prime numbers. We developed several conjectures. The first one is, there are only 3 consecutive numbers that are also $\tau_{(2)}$-primes (that is, there are no other triple of integers that are also $\tau_{(2)}$-primes), given by $(5, 6, 7)$. We also noticed that every integer is eventually a difference of $\tau_{(2)}$-primes. Furthermore, the $\tau_{(2)}$-primes are distributed among all equivalence classes modulo n, except possibly the equivalence class of n, where n is also a $\tau_{(2)}$-prime. This generalized the result from usual number theory, which says that the primes are distributed among all the equivalence classes that are invertible modulo n. We present examples and patterns that support our conjectures.

Keywords: prime distribution, primes

Automatic piano fingering data extraction from videos

Carlos J. Perez Vinelli, Department of Computer Science, University of Puerto Rico at Río Piedras.
Rafael Arce Nazario, Department of Computer Science, University of Puerto Rico at Río Piedras.

Research on predictive models to recommend piano fingerings from written music is an area of ongoing development in Machine Learning (ML). However, these models generally depend on vast amounts of data sets in order to produce an accurate model. Most of the research teams have to produce their own data sets due to the scarcity of datasets for developing these models. We are designing a simple yet effective method for automatically creating datasets for training fingering prediction models. The input to our system is a video/audio file of a pianist's hand recording of a musical piece, such as can be downloaded from YouTube. After capturing the video we extract the sound by using the ffmpeg tool and then deduce a MIDI file using Magenta (an ML library for manipulating sound). Information from the MIDI file, which essentially contains a sequence of (note, time) pairs, is used for extracting the video frames corresponding to the keys being played. We perform contour detection on an initial piano image using OpenCV to determine the position of each piano key. Then, for each captured frame we use OpenPose hand models to deduce the finger over the played key. The results of the automated fingering extraction will be compared against manually annotated data sets.

Epidemiological models for Chlamydia trachomatis in Puerto Rico

Frances M. Aponte Caraballo, Department of Biology, University of Puerto Rico at Cayey.
Genesis Rivera-Bachier, Department of Biology, University of Puerto Rico at Cayey.
Mayteé Cruz Aponte, Department of Mathematics and Physics, University of Puerto Rico at Cayey.

Sexual transmission infections (STIs) as Chlamydia trachomatis (CT) has increased in the Puerto Rican population and is the STI with the biggest prevalence in Puerto Rico. In this research work, we developed a dynamical system of differential equations based on an SIR type mathematical model. We use a SEIAT, to analyze the behavior of the Chlamydia transmission in the heterosexual female and male population between 15 35 years old. Doing a mathematical analysis, we determined if, in effect, Chlamydia is an epidemic and methods to decrease its prevalence. The experimental parameters from the developed model directly interfere in the behavior of the CT infection. Based on the numerical simulations we determined that the parameters we must manipulate for decreasing the infection are the probability of infection from an infected person to a completely susceptible population, the individuals that interrupts the treatment, individuals that have sexual intercourse during the treatment, days in recovery, changes in sexual partners and the treatment rate of symptomatic individuals.

Keywords: STI, Chlamydia trachomatis, epidemiologic mathematical model, prevalence, SEIAT, differential equation, simulation, epidemiology

Codes from Cayley Graphs

Fernando Piñero Gonzalez, Department of Mathematicas, University of Puerto Rico at Ponce.
Doel Rivera Laboy, Department of Mathematics, Pontifical Catholic University of Puerto Rico at Ponce.

In communications, we send messages through channels in which noise may cause errors to enter our message. If these errors occur, then how can we know what message was originally sent? This is where we use codes that are capable of error correction. One such family of error-correcting codes are LDPC codes. Low Density Parity Check (LDPC) Codes are linear codes constructed from sparse binary matrices. Most binary matrices chosen at random offer good decoding performance. However, LDPC Codes constructed from richly structured mathematical objects offer better decoding performance. This occurs because of the underlying structure the code will inherit from the object.

Our aim is to construct LDPC codes from different Cayley graphs in order to improve upon the performance of current codes, like those used in the Advanced Television Systems Committee 3.0. In our work, we study LDPC codes from Cayley graphs and their line graphs. We take advantage of this construction to develop a path-based decoder. We compare our LDPC code performance

with other codes in terms of error correction. We also impose further restrictions on the parity check equations to study other code rates and improve the minimum distance. We present some preliminary results.

Friend Factorizations

Ambar C. Rodríguez-Alicea, Department of Physics, University of Puerto Rico at Mayagüez.
Reyes M. Ortiz-Albino, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

The theory of τ-factorizations, defined in 2006 by Anderson and Frazier, can be interpreted as a restriction on the multiplicative operation of an integral domain. That is, a product of two or more elements is a τ-product, if the elements are pairwise related with respect to the relation τ. In 2014, Ortiz and Rivera study the behavior of the τ-factorization, using τ as the friendship equivalence relation over the nonzero non-unit integers, defined by $x\tau y$ if and only if $\sqrt{xy}$ is an integer. The definition of this relation appeared at the XXIII Colombian Math Olympiads. For the friendship relation, x can be multiplied with y if the square roots of their product xy is a perfect square. Ortiz and Rivera already studied some τ-factorization properties, but never tried to build a τ-irreducible τ-divisor graph for such framework. We present examples, patterns and some of the τ-irreducible τ-divisor graph of a nonzero non-unit integer.

Keywords: factorization, graph theory

Modeling ultrafine particulate matter through motor vehicles emissions and its effects as a trigger of asthma

Michelle N. Rosado Pérez, Bioengineering Graduate Program, University of Puerto Rico at Mayagüez
Karen R. Ríos Soto, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

The Global Burden of Diseases estimated asthma prevalence to be 339.4 million people worldwide in 2016 alone, being air pollutants one of its main causes. Ultrafine particulate matter (UFP) are air pollutants that affect the asthmatic population and as a consequence of its spread, exacerbate their asthma episodes. Ultrafine particles are particles with aerodynamic diameters of less than 0.1 μm that originate primarily from motor vehicles combustion. UFPs have not been well studied even though they are considered as the most dangerous type of particulate matter. On the other hand, many countries, including the United States and Puerto Rico, have no regulations that control their emissions. In order to study this problem, a mathematical model of nonlinear partial differential equations of reaction diffusion-advection type is introduced to show how the UFPs growth and disperse in the environment as a result of motor vehicle emissions, and in turn, how it affect an asthmatic population. Traveling wave solutions are assumed as an approach to solve and analyse the model, including a minimal speed of pollutants propagation. Equilibrium points for the system

are found, as well as the basic reproduction number R_p, that establish whether motor vehicles emissions will affect the asthmatic population. Numerical simulations were also performed, with parameter values obtained from the literature, which aid to identify which conditions are necessary to control the pollutants emissions and reduce the asthma episodes. This work confirms that the growth rate of pollutants emissions must be reduce in order to decrease the concentrations of UFPs in the environment. Similarly, the UFPs inhalation rate by humans should also be decreased to reduce the asthma prevalence. The use of filters on motor vehicles and face masks for the asthmatic population may be alternative methods of control to reduce UFPs emission and asthma episodes in the population, respectively.

Keywords: particulate matter, ultrafine particles, motor vehicles, asthma, epidemiological models.

Coarse-graining of Atomistic Models Using Neural Networks

Bernardo A. Roque-Carrión, Department of Mathematics, University of Puerto Rico at Humacao.
Lyxaira M. Glass-Rivera, Department of Mathematics, University of Puerto Rico at Humacao.
José O. Sotero-Esteva, Department of Mathematics, University of Puerto Rico at Humacao.

Classical Molecular Dynamics simulations contain the desired molecule as a solute in a solution. Simulating protein and solvent atoms requires many computational resources. One proposed method of reducing the computational expense of these simulations is a technique known as Coarse Graining, which reduces the total number of atoms to a smaller number of Coarse-Grained beads. In this project test a technique to derive the force fields in the Coarse-Grained Molecular Models from the all-atoms force field using Artificial Neural Networks. For the test, a small hairpin protein is used. Its corresponding coarse grain model atoms are the C atoms of the original molecule. A Neural Network is trained to compute the forces acting on the atoms of the Coarse Grain model. Energy measures of the all-atom and coarse grain models comparisons serve as measure of the accuracy of the coarse grained representation.

Unit Groups, Quotients, and new perspectives on τ_I-factorizations

Daniel A. Santiago, CROEM High School, Mayagüez Puerto Rico.
Reyes M. Ortiz-Albino, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

In 2011, Anderson and Frazier introduced the theory of generalized factorizations over integral domains, which is based on studying restrictions of the multiplicative operation in an integral domain. A special case of these constructions involves factorizations over the integers, where non-zero

non-unit integers are allowed to be multiplied if pairwise their difference is divisible by a given positive integer n. Using number theory and group theory, Hernández-Espiet (2019) characterized a structure of irreducible elements in such factorizations. This work comes as the latest in a sequence of different forms of approach to the problem, and is based on an algorithm using the structure of the group of units of the integers modulo the principle ideal generated by a positive integer n, denoted $U(n)$, and of a homomorphic image of $U(n)$, denoted $U'(n)$. In this work, we begin by generalizing the construction of the groups $U(n), U'(n)$ for arbitrary ideals in integral domains, and study the structure of the groups $U(n), U'(n)$ in the context of classical constructions in algebra such as Character Theory and Galois theory. Our results highlight the connections between the groups $U(n), U'(n)$, Cyclotomic Fields, and the proof of Quadratic Reciprocity.

Keywords: unit groups, galois groups, character groups

Non-parametric clustering diagnostic for k-means clustering solutions

Eliezer Santos-León, Department of Biostatistics and Epidemiology, University of Puerto Rico, Medical Sciences Campus.
Israel Almodóvar-Rivera, Department of Biostatistics and Epidemiology, University of Puerto Rico, Medical Sciences Campus.

Clustering in the presence of scatter can be challenging. Scatter (or outliers) observations in clustering are referred as observations that do not belong to any particular cluster. In this work, we proposed a distribution-free approach to assess clustering diagnostic. Our approach combines the speed and efficiency of k-means to find possible outliers in spherical structure groups. Our method uses the smooth estimation of the distribution function of the normed residuals from a given k-means solution and computes the nonparametric overlap between each pair of clusters. We proposed an estimation of the smoothing parameter, that combines the estimated overlap matrix and the number of groups. Then studied the proposed methodology in several simulation experiments with the presence of outliers. Our method is in general a top performer in finding spherical groups. The approach is also used to determine activated regions in a functional Magnetic Resonance Imaging study.

Keywords: k-means, clustering, outliers, nonparametric estimation

Mathematical Modeling of Vector-Borne Disease Dynamics with Environmental Factors

Yaria Sierra-Pérez, Department of Mechanical Engineering, Inter American University of Puerto Rico at Bayamón.
Carmen Caiseda, Natural Sciences and Mathematics Department, Inter American University of Puerto Rico at Bayamón.

The island or Puerto Rico has a mild tropical climate that promotes the positive growth of life and vector-borne diseases. Chikungunya and the Zika virus are examples of diseases that have affected our population. In this poster presentation a deterministic model that incorporates environmental factors is introduced. The model considers the human-vector interaction that spreads the disease. Model parameters are optimized to match real data of the 2015-2016 epidemics using a parameter estimation algorithm.

Keywords: deterministic model, disease dynamics, vector-borne diseases

On the Study of f-factorizations Over Unique Factorization Domains with $f(x) = x^r$

Geraldo E. Soto-Rosa, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.
Reyes M. Ortiz-Albino, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

An f-factorization is motivated by the notion of a τ-factorization, defined by D.D. Anderson and Andrea M. Frazier in 2011. We defined $a \cdot b$ to be an f-product if and only if a, b are non-zero non-unit elements such that $f(a) = b$ where f is a function; instead a symmetric relations as in the definition of Anderson and Frazier. This new concept does not coincide with the original one, except for the case of the identity function. Factorizations with respect to the identity function is very well understood, however, it is not the case for an arbitrary function f. We have been studied f-factorizations using an arbitrary polynomial function and the existence of nontrivial f-products in general. However, monomials of the form $f(x) = x^r$ provide an interesting behavior when considering an Unique Factorization Domain. Using properties of UFD's, we show the characterizations of what we define as f-atoms, f-primes and provide some examples.

Keywords: factorizations, unique factorization domain, monomials

Index