Contents

1 Charlas Plenarias / Plenary Talks

Divisibility and Factorization Theory

Felix Gotti, Massachusetts Institute of Technology.

In any algebraic structure with an underlying multiplicative monoid, an atom is, roughly speaking, an element that cannot be decomposed as the product of two non-invertible elements. Factorization theory studies the decompositions of elements into atoms, and the central role in this theory is played by finite sequences of atoms, which we call factorizations. In this talk, we will discuss several properties that are central in factorization theory, with the focus on the finite factorization property, which can be defined using only divisibility. An integral domain is called a finite factorization domain if every nonzero nonunit has only finitely many non-associate divisors. In addition, we will discuss two properties weaker than the finite factorization property, which can also be defined in terms of divisibility: they are the Furstenberg property and the IDF- property. With these properties in mind, we will discuss some current trends in factorization theory.

What do algebraists do

Irena Swanson, Purdue University.

Algebraists solve equations, which may sound boring, but they do it with clever and exciting methods. I will present my growth as a mathematician from the beginning, as well as solutions to cubic and quartic polynomials, Hilbert's Nullstellensatz, Hermann's constructive bounds, Gr obner bases, finishing with some still-open questions.

Graphs from rings

Ayman Badawi, American University of Sharjah.

In the last three decades, there has been considerable attention to graphs from rings and groups. In this talk, we will illustrate some connections between graphs' properties and rings structure.

2 Charlas Concurrentes / Concurrent Talks

(In alphabetical order using the last name of the speaker.)

Herramienta tecnológica de interconectividad para fomentar la proficiencia de estudiantes en el curso de geometría y mejorar el conocimiento matemático de futuros maestros

Jaime W. Abreu Ramos, Facultad de Educación, Universidad de Puerto Rico en Río Piedras.

Esta presentación ofrecerá detalles específicos de la herramienta de interconectividad utilizada en una investigación realizada en la Escuela Secundaria de la Universidad de Puerto Rico Recinto de Río Piedras. El estudio fue sufragado con fondos de la Fundación Nacional de Ciencias (NSF, por sus siglas en inglés) y aceptada para publicación en Journal of Mathematics Teacher Education en octubre del año 2021.

La charla explicará en forma breve el potencial de la herramienta tecnológica de interconectividad y su alcance para preparar a futuros maestros de matemáticas, así como poder fomentar la proficiencia de estudiantes en cursos de matemáticas. En particular serán ofrecidos contextos de educación en matemáticas donde la tecnología de interconectividad ofreció el espacio para fomentar positivamente el pensamiento matemático, comprensión conceptual y el razonamiento adaptativo para resolver problemas algebraicos y geométricos de nivel de escuela superior.

Kernel-estimated nonparametric diagnostic for outlier detection in a clustering framework

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Eliezer Santos Leon, Department of Biostatistics, University of Iowa.

Finding groups in the presence of scatter can be challenging. Scatters (or outliers) observations in clustering are referred to as those observations that do not necessarily belong or fit into any cluster. In this work, we are proposing a distribution-free approach to perform a diagnostic tool for a clustering solution to find potential outliers. Our approach uses a k-means solution to find the potential outliers in the homogeneous spherical groups. The method uses a smooth estimation of the distribution function of the normed residuals obtained from a given clustering solution. Further, we proposed a rule-of-thumb method to compute an estimate of the smoothing parameter for the estimation of the distribution function. In here we proposed a distribution-free approach to find an adaptive threshold for the detection of outliers. Our newly adaptive threshold can be computed by obtaining the quantile of the smooth estimate of the distribution function when difference of the tails of the smooth estimated distribution function with the tails of the empirical distribution function. Then, we studied our proposed diagnostic tool in several experiments with existing clustering algorithms that are suitable for the presence of outliers in a homogeneous spherical group. In general, our methodology is a top performer in finding the potential outliers in homogeneous spherical groups. Finally, we applied our distribution-free diagnostics to several datasets including

an experiment in a functional Magnetic Resonance Imaging to determine activated regions in a single-task experiment of finger-tapping.

Currículo de las Olimpiadas Matemáticas de Primaria en Iberoamérica

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Luis Cáceres Duque, Departamento de Ciencias Matemáticas, Universidad de Puerto Rico en Mayagüez.
Ariana Rodríguez Flores, Departamento de Ciencias Matemáticas, Universidad de Puerto Rico en Mayagüez.

En esta charla se discuten los resultados obtenidos en una investigación sobre el currículo o los temas principales que incluyen las diferentes pruebas realizadas en las olimpiadas de matemáticas de algunos países Iberoaméricanos, como lo son, Puerto Rico, Costa Rica, Argentina, Colombia, México, España, Bolivia, Nicaragua, El Salvador, Paraguay, Ecuador y Portugal. Para obtener dicha información, se realizó una búsqueda de exámenes de olimpiadas matemáticas de primaria de los distintos países, luego se analizó cada uno de los ejercicios de dichos exámenes, de manera que se pudiera clasificar cada uno en un área de la matemática específica (teoría de números, geometría, álgebra, patrones, aritmética, conteo, lógica, estadística y probabilidad) al que pertenece y a su vez se identifica un tema específico o la habilidad que un estudiante podría aplicar para la resolución del ejercicio.

An augmented multivariate hidden Markov model to capture dynamics in freely diffusing smFRET experiments

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Roberto Rivera, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

Enzymes serve a very important function as catalysts, accelerating chemical reactions. For this reason they are fundamental in biochemical processes. Single molecule Forster Resonance Energy Transfer (smFRET) experiments provide the capability of modeling the fluxional dynamics in enzyme conformations. Freely diffusing smFRET record the times photons detected by acceptor and donor fluorophore channels while diffusion occurs through a substrate with a confocal laser. This type of smFRET experiment does not limit the dynamic behavior of the molecule as the surface immobilized smFRET experiment does, but the recorded photons often come from the background and other aspects not related to the fluxional behavior. We propose an Augmented Multivariate Hidden Markov Model (AMHMM) to simultaneously model acceptor and donor channel photon counts while adding an auxiliary Markovian state that represents the enzyme not being in front of the laser. Initial parameter estimates are adjusted to extract information of the dynamic behavior of the molecule. Using simulated data, we show how our model is able to capture ground truth. Finally, we apply our AMHMM to model the dynamic behavior of an Alb3 enzyme.

Keywords: hidden Markov model, enzymes; smFRET; acceptor, donor

A characterization of uniform well-posedness for degenerate second-order abstract differential equations

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Carlos Lizama, Departamento de Matemática y Ciencia de la Computación, Universidad de Santiago de Chile.

We characterize the uniform well-posedness of the Dirichlet boundary value problem for the following degenerate second order abstract differential equation in Hilbert or Banach space X:

$$Mu''(t) + Au(t) = 0,\ t \in [0, \pi],$$

together with Dirichlet boundary conditions

$$Mu(0) = x_0 \text{ and } Mu(\pi) = x_\pi,$$

where x_0 and x_π are elements of X and A and M are closed linear operators on X.

Our characterization is given solely in terms of spectral properties of the data and uniform boundedness properties of an appropriate resolvent operator.

Keywords: degenerate second order equation, Dirichlet boundary value problem, uniform well-posedness

A new construction of few-weight codes via our GU codes

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Linear codes with few weights have applications in cryptography, association schemes, designs, strongly regular graphs, finite group theory, finite geometries, and secret sharing schemes, among other disciplines. Two-weight linear codes are particularly interesting because they are closely related to objects in different areas of mathematics such as strongly regular graphs, 3-rank permutation groups, ovals, and arcs. There exist techniques to construct linear codes with few weights, for example, the systematic exposition by Calderbank and Kantor. Ding et al. constructed few-weight codes using the trace function and Tonchev et al. generalized Ding's construction. In this talk, we present an elementary way to get two and three-weight codes from simplex codes and antipodal linear codes. An interesting application is the construction of uniformly packed linear codes from two-weight codes and quaternary quasi-perfect linear codes from three-weight codes.

Keywords: two and three-weight codes, simplex code, antipodal linear codes, and optimal additive code

Improving correlation properties of 3d periodic arrays constructed from elliptic curves

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Dorothy Bollman, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

Families of periodic 3D arrays are useful in various applications, including multiple target recognition, optical communications, and digital watermarking. A recent method consisting of composing a Legendre array plane with a sequence of row/column shift given by a cyclic group of points of an elliptic curve over a prime field. In general, this construction has good correlation properties to be implemented for digital video watermarks. But these properties can be improved by reducing the number of distinct differences in the shift sequence. We present theoretical and practical results on how to reduce the ratio of the peak autocorrelation to the non-peak auto- and cross- correlations by restricting the cyclic group to a certain class of elliptic curves.

Keywords: autocorrelation, cross-correlation, elliptic curve, watermarks

Creencias de los estudiantes de cálculo: un estudio inicial

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Kevin Palencia, Northern Illinois University.
Ricela Feliciano, Northern Illinois University.

Los cursos de cálculo tienen un rol importante en retener estudiantes a nivel universitario, en especial los estudiantes en ciencia, tecnología, ingeniería y matemáticas (STEM). En este estudio, presentamos los datos de una encuesta administrada a los estudiantes matriculados en cálculo 1, 2 o 3 en un recinto de la Universidad de Puerto Rico. El propósito de la encuesta es entender los puntos de vista de estos estudiantes acerca de la enseñanza del cálculo. Específicamente, estos datos demográficos nos permitieron hacer un análisis de las posibles correlaciones y diferencias entre las experiencias de los estudiantes en cursos de matemática de escuela secundaria y sus perspectivas acerca del aprendizaje y la enseñanza del cálculo a nivel universitario considerando diversas características demográficas. Reflexionaremos acerca de los problemas de retención de estudiantes en STEM y su posible relación a las prácticas actuales de la enseñanza del cálculo. En adición, discutiremos los hallazgos de este estudio y las posibles afinidades entre estas prácticas con las experiencias escolares de los estudiantes y sus expectativas.

Keywords: enseñanza del calculo, expectativas de los estudiantes, retención, STEM, tecnologías

On dihedral symmetric boolean functions: linear recurrences of exponential sums

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Luis A. Medina, Department of Mathematics, University of Puerto Rico at Río Piedras.

In this talk we present a study of exponential sums of Boolean functions that are fixed under the action of the Dihedral group. These functions are good candidates for some implementations in cryptography. We present a method to show that sequences of exponential sums of some families of Dihedral invariant Boolean functions are C-finite sequences.

Acknowledgements: The first author was supported, as a student, by The Puerto Rico Science, Technology and Research Trust under agreement number 2020-00124. The second author is supported, as a student, by Fondo Institucional Para la Investigación (UPRRP). The research of the third author was supported by The Puerto Rico Science, Technology and Research Trust under agreement number 2020-00124 and by the Fondo Institucional Para la Investigación (UPRRP). This content is only the responsibility of the authors and does not necessarily represent the official views of The Puerto Rico Science, Technology and Research Trust.

Keywords: exponential sums, dihedral group, C-finite sequences, balancedness

Initialization algorithm using distribution-free methods for clustering

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Israel Almodóvar, University of Puerto Rico at Mayagüez.

Clustering is an unsupervised technique that partitions a given dataset in homogeneous groups. A critical component in the performance of these clustering algorithms is the choice of the initial values. These values have a massive impact on the performance of these algorithms. In this work, we propose an initialization algorithm that combines the empirical likelihood approach with the normed residuals of the observations that have been chosen to be initial values and their cumulative distribution function. Potential candidates for the initial values are the farthest from each other. Based on the empirical likelihood these values will have a higher weight than those already considered. We prove if the initial values are obtained using our methodology the expected objective function is reduced. Simulation experiments were carried out to study the proposed methodology. Our methodology was compared with popular initialization methods in terms of performance, i.e., finding cluster solutions, as well in terms of iteration. Our methodology was a top performer in finding homogeneous spherical groups. Also, requiring a smaller number of iterations to converge than competing methods. Finally, we apply our methodology in a real dataset to identify the distinct regions of activation in a functional Magnetic Resonance Imaging study.

Tecnología de conectividad en la sala de clases para promover la construcción social del conocimiento matemático

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Las tecnologías usadas en el proceso de enseñanza y aprendizaje de las matemáticas, como las computadoras, programas de computadoras, aplicaciones, calculadoras, sistemas de conectividad, entre otros, brindan la posibilidad de manipular dinámicamente los objetos matemáticos en múltiples sistemas de representación, pero al mismo tiempo transforman el aula de clases en un espacio de interacción y participación de todos los estudiantes.

En esta charla se presentan algunos aspectos relacionados con el diseño, desarrollo, implementación y evaluación de actividades didácticas que integran tecnologías cuyos atributos promueven la construcción social del conocimiento matemático.

Se utilizó la Ingeniería Didáctica y la Educación Matemática Realista como marcos de referencia en una investigación realizada con estudiantes de octavo grado de una institución privada del área metropolitana de San Juan, Puerto Rico. Los resultados de la investigación muestran que la planificación cuidadosa de actividades con la calculadora gráfica, integrada a un sistema de conectividad en el aula de clases, promueve las discusiones y reflexiones de los estudiantes. Estas reflexiones dan una idea de cómo los estudiantes participan en la construcción social del conocimiento matemático. Además, los estudiantes demostraron que podían reflexionar sobre poderosas ideas matemáticas y adquirir conocimientos significativos.

Palabras clave: construcción social del conocimiento; conectividad en el aula; interacción social; conocimiento matemático

Ramsey like theorems on the rationals and some other structures

Peter Cholak, Department of Mathematics, University of Notre Dame.

Lets color substructures of the rationals of size 2 (there is just one such substructure) with 496 different colors. Then, it is known, that we can find an isomorphic substructure of the rationals $\widehat{\mathbb{Q}}$ where only 2 colors appear (among all substructures of size 2 within $\widehat{\mathbb{Q}}$). In fact, if we color substructures of the rationals of size n with 496 colors we can find isomorphic substructures of the rationals where the number of colors appearing is exactly the nth odd tangent number. So the rationals have *finite big Ramsey degree*. We will explore some other structures which have finite big Ramsey degree. Milliken's tree theorem plays a large role in showing these results. It turns out there is a complicated dichotomy between coding the halting problem or not based on the size of the subset. We will explore what this all means.

A universal robust bound for Bayes factors

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In this work we reformulate Smith and Spiegelhalter's Bayes Factor SSBF in terms of the evolving subject of Objective Bayes Factors. The SSBF was calculated and compared with the Intrinsic Bayes Factor for linear models, normal, exponential distributions and for separated models. We showed that the SSBF is a universal, feasible and useful bound for the Intrinsic Bayes Factors that changes with the amount of information. This bound is an improvement of the well known bound $-ep\log(p)$ proposed by Sellke et al. (2001) where p is the p-value.

Application of optimal control theory for pest management of coffee berry borers

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Since the detection of Coffee Berry Borers (CBB) *Hypothenemus hampei* in Puerto Rican coffee fields in 2007 efforts have been made to adapt integrated pest management techniques to control infestation levels. These biological controls include fungal pesticides, traps, and release of parasitoids, and are mostly directed at the CBB populations that have emerged from coffee berries. Moreover, these biological controls are expensive in terms of both labor and materials.

To study the impact of the biological controls on the CBB population we designed an optimal control problem that minimizes the infested coffee population and amount of control utilized subject to a system of ODEs that describes the dynamics of CBB infestation. The control was included as an additional mortality rate on the free CBB population. We used Pontryagin's maximal theorem to derive a set of adjoint equations, which were then solved numerically along with the ODE model using a forward-backward sweep and a fourth-order Runge-Kutta method.

The resulting optimized control shows similar behavior to the biological pest management strategies available. We observe that the first week, when coffee berries first appear, is of key importance and is when most of the pest management efforts should be concentrated. The optimal control reduced the amount of infested coffee berries at harvest time by about 58% when compared to the model without control.

Keywords: ordinary differential equations, nonlinear dynamics, optimal control, pest infestation, coffee agriculture, coffee berry borer

An efficient strategy to count cycles in the Tanner graph of quasi-cyclic LDPC codes

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David G. M. Mitchell, Klipsch School of Electrical and Computer Engineering, New Mexico State University.

In this talk, we present an efficient strategy to count the number of k-cycles, $g \leq k < 2g$, in the Tanner graph of a quasi-cyclic low-density parity-check (QC-LDPC) code with girth g using its polynomial parity-check matrix H. This strategy works for both (n_c, n_v)-regular and irregular QC-LDPC codes. We use some results on graph covers involving the images of cycles in the Tanner graph and the preimages of tailless backtrackless closed walks in the protograph to provide closed formulas for the number of k-cycles, $\mathcal{N}_k$, by just taking into account repetitions in some multisets constructed from the matrices $B_m(H) \triangleq \left(HH^{\mathsf{T}}\right)^{\lfloor m/2 \rfloor} H^{(m \mod 2)}$, where $m \geq 0$. Our approach has been shown to have low complexity. For example, in the case of QC-LDPC codes based on the $3 \times n_v$ fully-connected protograph, the complexity of determining $\mathcal{N}_k$, for $k = 4, 6$ and 8, is $O(n_v^2 \log(N))$, $O(n_v^2 \log(n_v) \log(N))$ and $O(n_v^4 \log^4(n_v) \log(N))$, respectively. The complexity, depending logarithmically on the lifting factor N, gives our approach, to the best of our knowledge, a significant advantage over previous works on the cycle distribution of QC-LDPC codes.

Acknowledgements: This material is based upon work supported by the National Science Foundation under Grant Nos. CCF-2145917, CNS-2148358, HRD-1914635, and OIA-1757207. A. G. F. thanks the support of the GFSD (formerly NPSC) and Kinesis-Fernández Richards fellowships.

Keywords: QC-LDPC code, Tanner graph, cycles, TBC walks

Large time behaviour for the heat equation on $\mathbb{Z}$, moments and decay rates

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En este trabajo estudiamos el comportamiento asíntotico de la solución de calor semi-discreta (en espacio) en la malla $\mathbb{Z}$

$$\begin{cases} \partial_t u(t,n) = \Delta_d u(t,n) + g(t,n), & n \in \mathbb{Z},\, t > 0, \\ u(0,n) = f(n), & n \in \mathbb{Z}, \end{cases} \tag{1}$$

donde $\Delta_d \phi(n) := \phi(n+1) - 2\phi(n) + \phi(n-1)$ es el laplaciano discreto. También se obtienen resultados de los momentos generalizados de la solución fundamental de (1) y se muestra que es consistente con el caso espacial-continuo.

Acknowledgements: This work was done in collaboration with L. Abadías (Universidad de Zaragoza), P. Miana (Universidad de Zaragoza), and J. C. Pozo (Universidad de Chile).

Keywords: Caputo fractional derivative; discrete fractional Laplacian; fundamental solutions; Bessel functions

Censored Zero-inflated Poisson Regression Models: Predicting Success in Undergraduate Math Courses

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Consider the number of times a student repeats a class until passing it. At the time of carrying out the data analysis, it is expected that there are still students who have not passed the course, so we do not know the number of times these students would repeat it until they pass, but we do know that this number is greater or equal to the observed value. In this case, these observations are censored. On the other hand, many students pass the class without repeating it, that is, there is a large percentage of zeros. In these cases, we will have censored zero-inflated count data. We establish statistical relationships between the Generalized Linear Models (GLMs) and the censored zero-inflated Poisson regression model, which allows us to find new likelihood equations. A thorough finite sample simulation study confirms the good behavior of the MLE using the new equations. Finally, we apply the model to a data set of students from the University of Puerto Rico to find models that predict if a student is at risk of failing introductory undergraduate math classes and how many times (on average) he or she may need to repeat the class.

Keywords: censored data; excess of zeros; generalized linear models; count data

La medida de Chermak-Delgado de los grupos de Heisenberg sobre $\mathbb{Z}_{p^n}$

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Para un grupo finito G y $H \leq G$, la medida de Chermak-Delgado de H (en G), que denotamos por $m_G(H)$, es el producto del orden de H por su centralizador. La medida de Chermak-Delgado de G es entonces $m^*(G) = \max\{m_G(H)|H \leq G\}$. Los subgrupos de G cuya medida es máxima constituyen un látice de subgrupos. Discutiremos una generalización del centralizador, y partiendo de esto, definiremos de forma análoga la pseudomedida de Chermak-Delgado de G, que también resulta ser un látice de subgrupos.

Utilizaremos esta generalización para demostrar que la medida de Chermak-Delgado para los grupos de Heisenberg sobre $\mathbb{Z}_{p^n}$ es p^{4n} para cualquier primo p.

Acknowledgements: Este trabajo fue realizado en colaboración con D. Allen (The City University of New York), S. Majewicz (Kinghsborough Community College), y M. Zyman (The City

University of New York).

La distribución de una probabilidad relativa a una medida

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Siguiendo la definición de caos, mezcla o incertidumbre de una probabilidad, según A. Uhlman, se presentan las propiedades de la distribución de probabilidad acumulada relativa a una medida arbitraria y métodos para el cálculo de esta función. Esta distribución es la base para la definición de la relación de incertidumbre o caos según Uhlman. Es una función cóncava, que además es convexa como funcional de la probabilidad y si se extiende a medidas positivas acotadas, es semicontinua inferiormente, entre otras propiedades que facilitan su cálculo. Se analizan también las propiedades para la densidad de probabilidad relativa a una medida que se derivan de las anteriores y de la variable aleatoria asociada con esta. En particular, se explora la densidad de probabilidad relativa de una probabilidad con respecto de otra y su posible uso como comparación de probabilidades. La prueba tradicional de chi-cuadrado se puede ver como una forma de comparación mediante la suma de cuadrados de las probabilidades relativas.

El entendimiento de la optimización de funciones de dos variables: un estudio usando esquemas en APOE

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Este estudio usa la teoría Acción-Proceso-Objeto-Esquema (APOE) para examinar el entendimiento de la optimización de funciones de dos variables. Se han hecho relativamente pocos estudios con APOE que se apoyan en la noción de esquema. Desde el punto de vista teórico, el estudio contribuye al mejor entendimiento de las nociones de esquema, la tríada para el desarrollo de esquemas, los tipos de relaciones entre componentes de un esquema, y su uso en una DG para modelar las construcciones mentales de los estudiantes.

Se propone una conjetura (llamada descomposición genética- DG) de construcciones mentales que los estudiantes pueden hacer para entender la optimización de estas funciones. La DG se puso a prueba usando entrevistas semiestructuradas con dos grupos de once estudiantes que recién habían terminado un curso introductorio de cálculo multivariable. Los estudiantes de un grupo usaron tareas diseñadas para ayudarles a hacer las construcciones en la DG y usando la estrategia pedagógica ACE (Actividades en grupos de tres o cuatro estudiantes, discusión en Clase, Ejercicios para el hogar). El otro grupo usó tareas del libro de texto y su enseñanza fue mayormente basada en conferencias.

Desde el punto de vista práctico, los resultados muestran que las actividades basadas en la DG fueron efectivas y sugieren cómo mejorar su potencial. Los resultados permiten mejorar la DG y sugieren dar más atención a la topología de los conjuntos dominio de las funciones a ser

optimizadas, a entender el alcance y limitaciones de algunos de los teoremas que comúnmente se aplican en el tema, y a la interpretación gráfica de estos teoremas.

Palabras clave: APOE, esquemas, optimización, funciones de dos variables, cálculo

Image classification using transfer learning and fine-tuning

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A proper training of a deep learning architecture may require a large dataset and powerful hardware, which is not always accessible. These days, several deep learning models that have been trained for image classification by big companies such as Google, OpenAI, or Meta are available for public use. Transfer learning and fine-tuning are methods that reuse models developed for a specific task to solve another one with minimal modifications. These methods significantly reduce the time, effort, and resources of the development cycle.

In this talk, I will present two examples of image classification using pre-trained models and discuss some considerations to complete the process successfully.

Keywords: image classification, deep learning, transfer learning, fine-tuning

Towards automatic bee re-identification with paint markings

Rafael Meléndez-Ríos, et al., Department of Computer Science, University of Puerto Rico at Río Piedras.

Bees are vital for agriculture, helping to pollinate various food crops that sustain human populations. Biologists have long studied them, both to better understand their behavior and protect them from threats such as parasites and the effects of climate change. Traditionally, to facilitate accurate identification of individual bees, bees are manually tagged with numbered tags, but such methods require specific manipulations and limits the number of bees that an experimenter can mark and track visually. Thus, the development of less invasive tagging methods as well as automatic tracking systems is crucial to increase the scale at which we can study individual behavior. In this work, we explore the use of paint marks as an alternative to tags for individual bee re-identification using deep learning models. The advantage of paint is that it is much more lightweight compared to tags which facilitates direct marking in the field during experimentations. We use a dataset of bee images created by using enamel paints of various colors to mark individual bees and train a residual convolutional neural network (ResNet50v2) on images from tracklets extracted with the pose detection library SLEAP. We train models across different train/test split protocols using contrastive learning with triplet loss, and present results for various evaluation methods. Results show promising re-identification performance for paint marks as an effective alternative to traditional tagging methods for reliable automated individual re-identification.

Acknowledgements: This research was done in collaboration with L. Meyers (Seattle University), J. Chan (University of Puerto Rico at Río Piedras/University of Central Florida), R. Mégret (University of Puerto Rico at Río Piedras), N. Fanfan (University of Puerto Rico at Río Piedras), J. L. Agosto-Rivera (University of Puerto Rico at Río Piedras), and T. Giray (University of Puerto Rico at Río Piedras). This work is supported by award #2021-67014-34999 from USDA-NIFA: "Deep-Pollinator: Enabling Large-Scale Video Analysis Of Pollinator Behavior With Deep Learning".

Keywords: deep learning, contrastive learning, re-identification, honey bee monitoring

Short cycle k-rotation Boolean functions and count

José E. Calderón Gómez, Department of Mathematics, University of Puerto Rico at Río Piedras.
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Rotation symmetric Boolean functions were introduced by Pieprzyk and Qu in 1999. They proved that these functions have efficient and secured cryptographic implementations. The class of rotation symmetric Boolean functions have been generalized to a class of functions known as k-rotation Boolean functions. These functions are useful, among other things, in the design of fast hashing algorithms with strong cryptographic properties. In this talk we give an explicit representation of generators of short cycles of k-rotation monomial Boolean functions. We use such representation to count the number of short cycles.

Acknowledgements: The first author was supported, as a student, by The Puerto Rico Science, Technology and Research Trust under agreement number 2020-00124. The second author is supported, as a student, by Fondo Institucional Para la Investigación (UPRRP). The research of the third author was supported by The Puerto Rico Science, Technology and Research Trust under agreement number 2020-00124 and by the Fondo Institucional Para la Investigación (UPRRP). This content is only the responsibility of the authors and does not necessarily represent the official views of The Puerto Rico Science, Technology and Research Trust.

Keywords: k-rotation boolean functions, short cycles, count

A numerical scheme that avoids the repulsion property in nonlinear elasticity: revisited

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For problems of the Calculus of Variations that exhibit the Lavrentiev Phenomenon, it is known that the *repulsion property* holds. That is, if one approximates the global minimizer in these problems by smooth functions, then the approximate energies will blow up. In SIDIM 2014 we showed

that the repulsion property holds in the context of nonlinear elasticity, in particular for problems in which the minimizers exhibit cavitation, and in SIDIM 2017 we reported some preliminary results on a numerical scheme that circumvents or works around the repulsion property. In this talk we give a full convergence result for this numerical scheme and report on some numerical simulations for radial problems in three dimensions, and for a problem in a fully two dimensional setting, on a disk but not assuming radial symmetry, for a class of materials called *elastic fluids*.

Keywords: nonlinear elasticity, cavitation, decoupling, elastic fluid

Are we there yet? An ECS pilot implementation for the Puerto Rico Department of Education

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Exploring Computer Science for Puerto Rico (ECS4PR) was the first project funded by the National Science Foundation to address the need for researching and integrating computer science education at the K-12 level in the Puerto Rico public school system. Since the official completion of ECS4PR's activities in 2021, the Puerto Rico Science, Technology and Research Trust has been supporting our efforts to scale-up the adoption and implementation of the curriculum "Explorando el mundo de la computación".

In 2022, the Puerto Rico Department of Education officially made a commitment to an ECS pilot implementation during the 2022 - 2023 school year. In order to achieve that goal, our team of researchers and facilitators offered several sessions of the first phase of the ECS professional development (ECS PD) to more than sixty teachers in the Spring of 2022. Currently, there are more than fifty teachers that are implementing or integrating ECS in schools from all seven OREs (i.e., Oficinas Regionales Educativas) and that are also participating in the second phase of the ECS PD.

In this work we will share: (a) the adaptations we made to the ECS PD model; (b) pre/post survey assessments of the ECS PD; (c) some statistics regarding teachers' perception about the ECS PD; (d) the overall impact in the school system; and finally, (e) since we are expecting to reach a wider community of teachers, one of our seasoned teacher facilitators will showcase some of her experiences and resources at implementing ECS at her school.

Acknowledgements: This material was initially based on work supported by the National Science Foundation under Grant No. 1738577 and currently is supported with funds provided by the Puerto Rico Science, Technology and Research Trust. Special thanks to Brenda Santiago for her excellent administrative work and for her invaluable contributions to each aspect of the project. This is a joint work with Michelle Borrero, College of Natural Sciences, Joseph Carroll-Miranda, Luis López Rivera, Agustín Corchado Vargas, College of Education, University of Puerto Rico at Río Piedras; Elizabeth Ríos, Enid Santiago, María Ortiz, Amabel Soto, Karol Ramírez, Norman Candelario, Puerto Rico Department of Education; Jorge Valentine, Frances Zenón, Puerto Rico Science Technology and Research Trust; and Milagros Bravo-Vick, External Evaluator.

Keywords: exploring computer science, ECS, professional development, computer science education

Implementación de guías para fortalecer debilidades conceptuales previas en estudiantes de Precálculo I de la UPRM bajo la modalidad de "Corequisite Support"

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En este trabajo se presentan los resultados obtenidos durante la implementación de unas guías de refuerzo previo (basadas en la metodología de "Corequisite Support") que fueron creadas durante la primera fase de esta investigación. El objetivo primordial es mitigar las debilidades conceptuales (conceptos que debería conocer un estudiante antes de someterse a un curso en particular), presentes en los estudiantes de nuevo ingreso de la Universidad de Puerto Rico, Recinto de Mayagüez (UPRM), al curso de Precálculo I.

La implementación de esta fase se llevó a cabo durante el segundo semestre 2021 – 2022, con la participación de 5 docentes quienes se encontraban impartiendo el curso de Precálculo I en la UPRM. La ejecución de esta investigación tuvo lugar de manera paulatina (semanal) conforme se iba desarrollando el semestre en cuestión, a modo que los estudiantes recibían el refuerzo previo semanal antes de ver los conceptos fundamentales para los que era útil cada refuerzo.

Cada docente organizó en sus grupos de trabajo un grupo control (sin la metodología) y un grupo experimental (con la metodología), con el fin de poder observar las diferencias entre los grupos. Para tal fin se implementó una comparación de medias entre los grupos haciendo uso del estadístico t de student; bajo un diseño anidado o jerárquico.

Palabras clave: precálculo I, debilidades conceptuales, conceptos previos, grupo control y experimental, refuerzo previo, corequisite support

Permutation Invariant Parking Assortments

Eric J. Pabón Cancel, et al., Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

We introduce *parking assortments*, a generalization of parking functions in which there are $n \in \mathbb{N}$ cars of assorted lengths $\mathbf{y} = (y_1, y_2, \ldots, y_n) \in \mathbb{N}^n$ entering a one-way street containing $m = \sum_{i=1}^{n} y_i$ parking spots. The cars have parking preferences $\mathbf{x} = (x_1, x_2, \ldots, x_n) \in [m]^n$, where $[m] := \{1, 2, \ldots, m\}$, and enter the street in order. Each car $i \in [n]$, with length y_i and preference x_i, follows a natural extension of the classical parking rule: it begins looking for parking at its

preferred spot x_i and parks in the first y_i contiguously available spots thereafter, if there are any. For a fixed $\mathbf{y}$, if all cars are able to park under the preference list $\mathbf{x}$, we say that $\mathbf{x}$ is a *parking assortment* for $\mathbf{y}$. In this way, parking assortments generalize parking functions, which correspond to the case in which $\mathbf{y}$ is a list of all ones. Moreover, for a fixed $\mathbf{y}$, the set of parking assortments contains the set of *parking sequences*, which correspond to the case in which car i fails to park if, upon its arrival, the first available spot $j \geq x_i$ is such that either $j + y_i - 1 > m$ or at least one of the subsequent spots $j + 1, \ldots, j + y_i - 1$ is already occupied.

We say a parking assortment $\mathbf{x}$ for $\mathbf{y}$ is *permutation-invariant* (or *invariant* for short) if all of the rearrangements of $\mathbf{x}$ are parking assortments for $\mathbf{y}$. While all parking functions are invariant, this is not the case for parking assortments in general. The goal of this work is to provide necessary and sufficient conditions for parking assortments to be invariant for a fixed arbitrary $\mathbf{y}$. While obtaining a full characterization for an arbitrary number of cars remains elusive, we do so for the case of two and three cars. Given the technicality of these results, we introduce the notion of *minimally invariant* car lengths, for which the only invariant parking assortment is the all ones preference list. We provide a concise, oracle-base characterization of minimally invariant car lengths for any number of cars and derive certain closure properties of this set. We conclude with a conjecture regarding a Boolean formula for minimally invariant parking assortments with four cars.

Acknowledgements: This work was done in collaboration with D. Chen (Johns Hopkins University), G. Sargent (University of Notre Dame), P. E. Harris (University of Wisconsin-Milwaukee), and J. C. Martinez Mori (Cornell University). This material is based upon work supported by the National Science Foundation under Grant No. DMS-1929284 while the authors were in residence at the Institute for Computational and Experimental Research in Mathematics, Brown University.

Keywords: parking functions, combinatorics

Within-host bayesian joint modeling of longitudinal and time-to-event data of *Leishmania* infection

Félix M. Pabón-Rodríguez, Department of Biostatistcs, The University of Iowa College of Public Health.

The study of communicable diseases requires serious attention from researchers and health professionals at all scales, from infection prevention to individualized care for patients. In this talk, we will focus on *Leishmania infantum*, a microbe of great public health concern, and present a Bayesian model for the within-host dynamics of the infection using longitudinal data collected from a cohort of dogs naturally exposed to *Leishmania infantum*. We consider the interplay between key drivers of the disease process, inflammatory and regulatory responses of the immune system, the role of co-infections and preventative treatment. Because several subjects developed a severe case of the disease and died as a result of progressive *Leishmania* infection, the model also characterizes the relationship features of the longitudinal outcomes and time of death. This analysis has direct clinical and preventive relevance, presenting possible opportunities for application in veterinary practice and exposure prevention. Results from this work may support future efforts to reduce the risk of human exposure to *Leishmaniasis*.

Keywords: bayesian modeling, co-infection, immune responses, visceral Leishmaniasis, detection limit, censoring, longitudinal data, survival, forecasting

A General Bayesian Theory of Hypothesis Testing and Model Selection

Luis Pericchi Guerra, et al., Department of Mathematics, University of Puerto Rico at Río Piedras.

This talk gives an introduction and recent developments in Bayesian hypothesis testing and model uncertainty. The focus is on a methodology that is, in principle, general and which can be automatically applied. This thus falls within the domain of objective Bayesian analysis. The key concepts of Bayesian hypothesis testing are introduced and illustrated, including a discussion of common mistakes in choosing prior distributions. A specific prior distribution (the intrinsic prior) is recommended for testing any point null hypothesis; it is a relatively simple function of the null and alternative models. Finally, testing when there are nuisance parameters is discussed. Then we introduce the basics of dealing with model uncertainty. The choice of prior probabilities of models is discussed, with emphasis on the default choice that adjusts for multiple testing. Topics such as model averaging, prediction, and the median probability model are also discussed. The key difficulty in model uncertainty is is that of choosing prior distributions for model parameters, especially when there are many models involved. We focus on the development of intrinsic priors and expected posterior priors. These are our recommended approaches to the development of automatic priors because they are extremely general and have excellent properties. There are a number of software solutions, in the form of R packages, that make the application of Bayesian tools for testing and model uncertainty extremely easy, like in "BayesVarSel". Bayesian testing and model uncertainty is a huge methodological area in statistics, and many important techniques and approaches will be mentioned herein only briefly. This talk is based on a Chapter in the Handbook of Statistics by the same authors.

Acknowledgements: This work is done in collaboration with Berger J.O., Garcia-Donato G. and Moreno E.

Improving the minimum distance bound of Trace Goppa codes

Fernando L. Piñero, et al., Department of Mathematics, University of Puerto Rico at Ponce.

In this talk we show that Goppa codes with Goppa polynomial $g(x) = \mathbf{Tr}_{\mathbb{F}_{q^m}\setminus\mathbb{F}_q}(x)$ where $m \geq 3$ has a much better minimum distance than $d \geq 2\deg(g(x)) + 1$. In several cases, this Goppa code construction beats the corresponding BCH code construction.

Acknowledgements: This work was done in collaboration with I. Byrne (Virginia Tech), N. Dodson (Middlebury College), R. Lynch (Notre Dame University), and E. Pabón–Cancel (University of Puerto Rico at Mayagüez).

Keywords: Goppa codes, distance bound, subfield subcodes

Música, ciencia y matemáticas: un curso introductorio multidisciplinario de gran belleza e impacto

Arturo Portnoy, Departamento de Ciencias Matemáticas, Universidad de Puerto Rico en Mayagüez.

En esta charla describiré un curso diseñado por tres profesores de la Universidad de Puerto Rico en Mayagüez: un físico, una teórica de la música y un matemático. Hablaremos sobre sus objetivos, el diseño del curso, y las experiencias enseñándolo durante dos semestres, con una audiencia muy diversa, pues es un curso sin prerequisitos. Discutiremos algunos de los temas más fundamentales e interesantes, y mostraremos actividades interactivas que se proponen en el curso y que serán parte de un libro que en un futuro cercano será publicado. Hablaremos sobre la sinergia de la multi/interdisciplinariedad, de la necesidad que los estudiantes trabajen en grupo y que sean capaces de producir informes y comunicaciones profesionales, parte de los objetivos del curso.

El curso propone abordar temas diversos, difíciles, y conectar entre disciplinas sin pretender entendimiento detallado y profundo, más bien lograr conexiones y que los estudiantes puedan ver el gran panorama: como las disciplinas no se desarrollan aisladas, sino que una alimenta a la otra; la historia está llena de estas interacciones e influencias entre disciplinas. Hablaremos brevemente sobre los retos presupuestarios y administrativos que presupone una iniciativa como esta, retos que finalmente terminaron poniendo en pausa el ofrecimiento del curso.

Arte Generativo: el código y la matemática como medio creativo

Elio Ramos-Colón, Departamento de Matemática, Universidad de Puerto Rico en Humacao.

El arte generativo combina elementos de la ciencia de cómputo, la matemática, el arte, y el diseño. Generalmente se define como el proceso de crear piezas de arte en donde se utiliza total o parcialmente un sistema autónomo como lo puede ser una computadora o un artefacto mecánico. La misma requiere el uso de tecnologías, modelos matemáticos, algoritmos y lenguajes de programación especializados. En esta charla daremos una visión panorámica del arte generativo desde la perspectiva de un científico de cómputo que, en los últimos años, ha realizado varios experimentos y proyectos en esta área. Se argumentará que el arte generativo permite divulgar la importancia y la aplicabilidad de las matemáticas más allá del ámbito científico.

Palabras claves: arte generativo, matemática y arte, programación creativa

Searching new criteria for absolute irreducibility of polynomials over finite fields

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Moises Delgado, Department of Mathematics-Physics, University of Puerto Rico at Cayey.

A multivariate polynomial defined over a finite field K is irreducible if it cannot be expressed as the product of nonconstant polynomials. Otherwise, it is reducible. However, some of them are irreducible in K, but they are reducible in some extension Km of K. There exists multivariate polynomials defined over K that are irreducible in any extension of it. These types of polynomials are called absolutely irreducible polynomials. Finding criteria to test absolute irreducibility is fundamental for applications in pure and applied mathematics as algebraic geometry, combinatorics, coding theory, cryptography, finite geometry, among others; and not much simple criteria are available in the literature. In this talk we introduce the concept of the Degree Gap of a polynomial, and using recent results from Agrinsoni, Delgado and Janwa, we will propose new criteria for testing absolute irreducibility of infinite families of multivariate polynomials defined over finite fields of characteristic 2.

Keywords: absolute irreducibility, homogeneous polynomial, square free polynomial, degree gap

Application of the concept of a mathematical function to the management of diagnostic graphs of human proteins

Julio Eric Rodríguez Vargas, Department of Mathematics, University of Puerto Rico at Aguadilla.

In clinical diagnosis, there are several types of analysis that give the Doctor the ability to inspect the amounts of proteins in human fluids to determine the health of the patient. Within this variety of clinical procedures, electrophoresis stands out, which is an analysis methodology that allows the clinician to break down a sample into various constituent fractions, making them visible through a graph.

There are many diagnostic equipments on the market that can do this type of analysis, however, our focus was on how to operate all the details of the system management and how to bring the results to the Physician in the most efficient way (resources and time) using technology and a simple Mathematical concept.

I will be presenting how we apply the mathematical concept of a function to extract, process, compress and transmit all the information related to the Electrophoresis graphs. The large-scale goal was to apply the "zero value" paradigm to achieve a reduction of the COO (cost of ownership) of the system.

Acknowledgements: LabInfo LLC, Bionuclear Puerto Rico, Inmuno Reference Lab.

Keywords: mathematical function, cost of system ownership, cost of system operation

Approximation of the Local Bayes Empirical Factor for the comparison of a $DLMARMA(1,1)_\tau$ and a $DLMARMA(1,0)_\tau$ models by introducing the evolution matrix $G_\tau = G - \tau I$ for sufficiently small τ

Angélica M. Rosario Santos, Department of Mathematics, University of Puerto Rico at Rio Piedras.
Luis Pericchi Guerra, Department of Mathematics, University of Puerto Rico at Rio Piedras.

In this talk, we present an alternative to obtaining an approximation of the Local Bayes Empirical Factor $B_{j,i}$ (based on real training data) for the comparison of the Nested Dynamic Linear Models (DLM's) $M_j = DLM \quad ARMA(1,1)_\tau$ and $M_i = DLM \quad ARMA(0,1)_\tau$.

The motivation of our research project is to have a robust Bayesian way to select between $ARMA(p,q)$ models for migration trends. Due to the complex behavior of the migratory component and its dependence on economic and climatic factors, a dynamic modeling seems more reasonable. Hence we consider the most widely used DLM representation for a $ARMA(p,q)$ model.

In 2001, Rodríguez and Pericchi developed a specific theory to define local Intrinsic Bayes factors (IBFs) for dynamic linear models based on simulated replicates of the initial set of observations. When the variance is known, they provide exact formulas for its calculation that require the specification of a non-singular evolution matrix G. However, the dynamic linear representation of a model $ARMA(p,q)$, has G singular when $p \leq q$. Therefore, by introducing a perturbation τ we define $G_\tau = G - \tau * I$ where G_τ is now invertible for sufficiently small τ . An algorithm for choosing the smallest possible τ^* to approximate $B_{j,i,\tau}$ is applied. Finally, an example was done with data generated from M_j for show the pattern of convergence of $B_{j,i,\tau}$ as τ approaches τ^*.

Keywords: Bayes Factors, DLM ARMA Models

Long cycles for spread spectrum - *from Hedy Lamarr to Oscar Moreno*

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Sequences and arrays play an important role in a variety of applications in digital communication. Berlekamp and Moreno, working over "extended" fields of characteristic 2, $\mathbb{F}_{2^n} \cup \{\infty\}$, constructed linear fractional transformations that permute the "extended" field in a single cycle of length 2^n+1. This provides an algebraic construction of sequences that can be applied to questions on spread spectra and applications to multiple-target, digital watermarking and optical orthogonal codes, among others.

The Berlekamp-Moreno result was erroneously extended to any characteristic p and the result echoed in several papers. One purpose of this work is to correct the erroneous generalization. We also present a complete picture, in both even and odd characteristics, of the cycle lengths obtainable from linear fractional transformations.

Dynamic time warping and energy clustering for brain activation under stimuli in fMRI study

Eliezer Santos Leon, Department of Biostatistics, University of Iowa.

Functional magnetic resonance imaging (fMRI) is a technique used to generate sequential images of brain activity. It has been widely applied to study changes in brain function due to neurological conditions and psychological disorders. For fMRI studies, the blood oxygenation level dependent (BOLD) is commonly measured to map change in blood flow with a neural activity of interest. Although BOLD contrasts can give insight into brain activity through indirect estimates, it has some drawbacks. One of them is that cerebrospinal fluid movement and circulatory system can influence the measurements, thereby adding noise to the system. Using data from an audiovisual study, this project implements the use of distance measures to capture signal dissimilarities which leads to the identification of activated regions associated with audiovisual stimuli.

Keywords: clustering, fMRI, BOLD, audiovisual stimuli, time series, dynamic time warping, energy distance

Diffusion on bronchial trees I: construction and approximation results

Kevin Silva-Pérez, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez. *Alejandro Vélez-Santiago,* Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

We investigate a class of regions with fractal ramified boundaries, which are a idealization of the bronchial trees in $\mathbb{R}^2$. Following the approach of Y. Achdou and N. Tchou, we will provide a construction of the ramified domains Ω_a with ramified boundary Γ_a^∞, for a a parameter with $1/2 \leq a \leq a^* \simeq 0.593465$. We then establish several properties for the sets Ω_a and Γ_a^∞, and prove in particular that Ω_a is a 2-set, and that Γ_a^∞ is a d-set for $d := -\log(2)/\log(a)$. At the end, motivated by some methods employed by Jia, we construct approximating sequences $\{\alpha_n\}$ and $\{\eta_n\}$ for the Hausdorff measure $\mathcal{H}^d(\Gamma_a^\infty)$ of the fractal boundary Γ_a^∞, in the sense that

$$\alpha_n \leq \mathcal{H}^d(\Gamma_a^\infty) \leq \eta_n \quad \text{and} \quad \alpha_n \nearrow \mathcal{H}^d(\Gamma_a^\infty) \swarrow \eta_n.$$

In this way, we provide a way to approximate the length of bronchial trees in the lungs system.

Keywords: Hausdorff measure, Hausdorff dimension, d-sets, self-similarity, open set condition, (ϵ, δ)-domains

Computable finite factorization domains

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In the ring of integers $\mathbb{Z}$ we can obtain the set of all the divisors of a given number using the Division Algorithm. Moreover, because there are finitely many divisors for each element, the Kronecker Method allows us to construct an algorithm to find the set of all divisors for polynomials in $\mathbb{Z}[x]$. In both cases, the existence of these algorithms implies that the set of irreducible elements is computable relative to the copy of the structure. Other integral domains, like the quadratic extensions of $\mathbb{Z}$, also have this property. In general these domains are called *Strongly Computable Strong Finite Factorization Domains* (SCSFFD).

In 2017, these structures were used to prove the existence of an integral domain where the set of irreducible elements is computable while the set of prime elements is not. Although differences are known between irreducible and prime elements in the algebraic context, this result shows how different they can be in computability context. Also, the authors provided conditions for which a Unique Factorization Domain (UFD) A is a SCSFFD. However, this is not sufficient because there exists integral domains like $\mathbb{Z}[\sqrt{-5}]$ that are SCSFFD but not UFD.

Our work completely classifies SCSFFD's in general by showing the existence of a computable norm that possess certain properties as being able to solve norm-form equations computably and allow us to extend the notion of strongly computability to Finite Factorization Domains in general. We use the technique of constructions by priority requirements and examples using quadratic extensions are provided using the algorithmic properties of the *Pell*'s equation $x^2 \pm Dy^2 = N$.

Keywords: computable structures, quadratic extensions, computable norms, finite factorization domains, norm-form equations

Un modelo Markoviano multivariado y el uso de bootstrapping para capturar incertidumbre

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Las propiedades presentes en los nanomateriales son altamente dependientes de su tamaño, forma y composición atómica, lo que genera que posean características ópticas, electrónicas y magnéticas, diferentes y no bien conocidas. Predecir o cuantificar con cierto grado de confianza la dinámica de la configuración atómica de las nanopartículas durante procesos de difusión superficial en experimentos in situ, es de gran interés porque nos permite conocer de antemano los cambios en su estructura atómica con la suficiente certeza para hacer un uso eficiente de sus propiedades y maximizar sus beneficios. Proponemos un modelo multivariado oculto de Markov (MMOM) para caracterizar la dinámica de las intensidades de las columnas atómicas de una nanopartícula durante un experimento TEM. Dichas intensidades se extraen utilizando un algoritmo de blob detection que nuestro equipo desarrolló recientemente. Primero demostramos el rendimiento de MMOM utilizando datos

simulados. Luego aplicamos el MMOM a datos de óxido de cerio y calculamos la incertidumbre de los estimados usando bootstrapping.

Analytical and data-driven wave approximations of an extended Schrödinger equation

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Aaron Phillips, Department of Mathematics, Lamar University.
José M. Vega–Guzmán, Department of Mathematics, Lamar University.

In this work we use both analytical and numerical techniques to discuss wave solutions within the framework of an extended nonlinear Schrödinger equation with constant coefficients equipped with spatiotemporal dispersion, self-steepening effects, and a Raman scattering term. We present the exact traveling wave solution of the system in terms of Jacobi elliptic functions and mention some symmetry results as they relate to the resulting ordinary differential equation. A constructed bright soliton solution serves as the base to compare a numerical solution of the system using spectral Fourier methods with a precise statistical low-rank approximation using a data-driven approach aided by the Koopman operator theory. We found that the spatiotemporal feature added to the model serves as a regularizing tool that enables a precise reconstruction of the original solution.

Acknowledgements: The authors acknowledge the effort, guidance, comments, advise and patience from all faculty members involved in the REU@LU 2021. This study was supported in part by the Lamar University Research Enhancement Grant #420266, and by the National Sciences Foundation (NSF Award # 1757717).

Keywords: Schrödinger equation; traveling waves; integrability; Koopman operator theory, data-driven techniques, low rank approximation

From p-values to posterior probabilities of null hypothesis

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Minimum Bayes factors are commonly used to transform two-sided p-values to lower bounds on the posterior probability of the null hypothesis, as in Vovk (1993) and Sellke et al. (2001), where the bound $-e \cdot p \cdot \log(p)$ is presented. This bound is easy to compute and explain; however, it does not behave as a Bayes Factor. For example, it does not change with the sample size. This is a very serious defect, particularly for moderate to large sample sizes, which is precisely the situation in which p-values are most problematic. In this article, we propose adjusting this minimum Bayes factor with the information to approximate an exact Bayes factor, not only when p is a p-value but also when p is a pseudo-p-value in the sense of Casella and Berger (2001, p.397). Additionally, we

develop a version of the adjustment for linear models using the recent refinement of Prior Based BIC, Bayarri et al.(2019).

Keywords: p-value calibration, Bayes factor, linear model, pseudo-p-value, adaptive levels

Diffusion on bronchial trees II: solvability and global regularity results

Kevin Silva-Pérez, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.
Alejandro Vélez-Santiago, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

We consider a domains $\Omega_a \subseteq \mathbb{R}^2$ with ramified boundary Γ_a^∞ constructed by Y. Achdou and N. Tchou, for a a parameter with $1/2 \leq a \leq a^* \simeq 0.593465$. This domain represents an idealization of bronchial trees in the lungs system. Since the exchanges between the lungs and the circulatory system take place only in the last generation of the bronchial trees, an accurate model for diffusion of oxygen may involve inhomogeneous Robin boundary conditions over Γ_a^∞. Therefore, we investigate the realization of the diffusion equation

$$\frac{\partial u}{\partial t} - \Delta u + \alpha u \,=\, f(x,t) \qquad \text{in } \Omega_a \times (0,\infty)$$

with mixed boundary conditions

$$\frac{\partial u}{\partial \nu_\mu} + \beta u \,=\, g(x,t) \text{ on } \Gamma_a^\infty \times (0,\infty); \quad , \quad u = 0 \text{ in } (\partial\Omega_a \setminus \Gamma_a^\infty) \times (0,\infty),$$

and $u(x,0) = u_0 \in C(\overline{\Omega}_a)$, where $\frac{\partial u}{\partial \nu_\mu}$ represents a generalized notion of a normal derivative over irregular surfaces, $\alpha \in L^r(\Omega_a)^+$, $\beta \in L^s_\mu(\Gamma_a^\infty)^+$ with ess $\inf_{x\in\Gamma_a^\infty} |\beta(x)| \geq \beta_0$ for some constant $\beta_0 > 0$, where $\min\{r,s\} > 1$. We show unique solvability of this diffusion equation, and moreover we establish that weak solution of this model equation are globally continuous in space and in time.

Keywords: ramified domains, Robin boundary conditions, weak solutions, Schauder estimates

Aprendizajes de un proyecto de formación de maestros de matemáticas de nivel secundario

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Omar Hernández Rodríguez, Universidad de Puerto Rico en Río Piedras.

Presentamos los logros de un proyecto del programa Improving Undergraduate STEM Education (IUSE) de la National Science Foundation. Incluiremos, además de los logros programáticos, los aprendizajes como investigadores principales del proyecto en las diferentes etapas. El objetivo principal del proyecto era desarrollar el conocimiento de los futuros maestros de matemáticas (FMs) sobre el uso de las herramientas tecnológicas de interconectividad para que aporten a sus estudiantes un mejor entendimiento y aprendizaje de las matemáticas. Para lograr este objetivo, el

proyecto adoptó una estrategia denominada estudio de lecciones, que fue diseñada para mejorar la enseñanza a través de ciclos de estudio, planificación, enseñanza y reflexión. En el estudio de lecciones participan colaborativamente los FMs, los profesores universitarios de los FMs y los maestros cooperadores (los maestros en servicio con experiencia con los que los FMs tienen sus experiencias clínicas iniciales). Como resultados reportamos:

1. Modificación del curso de metodología de enseñanza de las matemáticas en el nivel secundario para incorporar el uso de Desmos Classroom para fomentar discusiones productivas en el salón de clases.

2. Adopción de una metodología que reconoce la importancia del trabajo en grupo y la aportación de todos los miembros del equipo compuesto por investigadores, asistentes de investigación, maestros cooperadores y futuros maestros.

3. Investigaciones del proceso de aprendizaje de los futuros maestros y maestras de matemáticas y de sus maestros y maestras colaboradores.

4. Publicación de artículos en revistas arbitradas y libros editados.

5. Presentación en conferencias nacionales e internacionales.

6. Divulgación en medios electrónicos.

Los resultados nos proveyeron información para contribuir a cerrar la brecha que existe entre los cursos de métodos y las prácticas clínicas. Los próximos pasos apuntan a la posibilidad de expandir los resultados a los cursos de ciencias y de matemáticas a nivel elemental.

Reconocimientos: Este proyecto fue auspiciado por la National Science Foundation, Award # 1930950 - Developing Technological Pedagogical Content Knowledge of Pre-Service Math Teachers.

3 Afiches / Posters

(In alphabetical order using the last name of the presenter.)

Machine learning approaches to study death incidence by race in colorectal cancer patients after treatments

Frances M. Aponte-Caraballo, School of Public Health, University of Puerto Rico-Medical Sciences Campus.
Frances Heredia-Negrón, enter for Collaborative Research in Health Disparities, University of Puerto Rico-Medical Sciences Campus.
Abiel Roche-Lima, enter for Collaborative Research in Health Disparities, University of Puerto Rico-Medical Sciences Campus.

Colorectal Cancer (CRC) cases have increased worldwide. In USA, African Americans have a higher incidence than other races. Machine Learning (ML) approaches are used extensively in medical sciences, for example to predict diagnoses and treatments. In this research, we aim to use ML to study specific factor or variables affecting the high incidence of CRC mortality by race after receiving treatments, as well as creating models to predict death.

Data: Metastatic CRC Genes Sequencing Study, 2018 (TCGA). The patient's inclusion was based on receiving chemotherapy and grouped by race (White-American and African-American). ML approaches: Five supervised ML methods were implemented (Logistic-Regression, Linear-Discriminant-Analysis, Classification-Regression-Trees, K-Nearest-Neighbor, Naive-Bayes). Correlation analysis, feature extraction and predicting algorithms were used. A Mini-Batched-Normalized-Mutual-Information-Hybrid-Feature-Selection method was used to extract features including more than 25,000 genes.

The best model was obtained with the Classification-Regression-Trees algorithm (AUC-ROC= 0.91 for White-American, AUC-ROC=0.89 for African American). The features "DBNL gene", "PIN1P1 gene" and "Days-from-birth" were the most significant variables associated with CRC mortality for White-American, while "IFI44L-gene", "ART4-gene" and "Sex" were the most relevant related to African-American.

ML approaches are applied to identify factors associated with CRC mortality among more than 25,000 variables. ML-based models were obtained to predict death after CRC chemotherapy treatment by race. These factors and models are promising for further analysis and decision-making tools to study CRC from a precision medicine perspective for minority health.

Acknowledgements: This research was supported by CAPAC (Award Grant Number# R25CA240120) from the NCI. RCMI grant U54 MD007600 from the NIMHD-NIH.

Keywords: colorectal cancer, machine learning, mortality, health disparities, race

A conservative splitting-high order finite difference method for coupled Gross-Pitaevskii equations in 2D

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An energy conservative field directional scheme to approximate the solution of Gross-Pitaevskii systems of nonlinear equations in 2D is derived from the general idea of the Lie-Trotter splitting technique. In addition, the method preserves a numerical invariant which can be interpreted as a time step perturbation of the total mass. Under certain conditions, e.g., when the Josephson junction is neglected or when the initial conditions satisfy certain algebraic properties, mass conservation is also achieved. Using a suitable class of high order symmetric finite difference approximations of the Laplacian operator in 2D, we prove that, for the thermodynamically stable regime, the directional splitting scheme is of order $\tau + h^{2p}$ in the $l_{2,h}$-norm, for $p = 1, 2, 3, 4$. High order accuracy in time can be achieved by combining the proposed basic time step with well known composition methods, as a result conservative methods of order $\tau^{2q} + h^{2p}$ with $q = 1, 2, 3, 4$ are obtained. Conservation and accuracy are numerically validated for model problems with and without internal atomic Josephson junction. Our performance study shows that the proposed technique is suitable for long-term simulations.

Keywords: mass and energy conservation, Gross-Pitaevskii equations, high order finite difference method, composition methods

Time series forecasting supported by fractal-based classes

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Clara E. Isaza, Department of Biology, University of Puerto Rico at Mayagüez.
Mauricio Cabrera-Ríos, Department of Industrial Engineering, University of Puerto Rico at Mayagüez.

In time series analysis, forecasting is understood as generating a prediction based on known previous records. This process is important in all STEM fields.

This work proposes to approach forecasting using classes defined by a fractal scheme. An initial time series is configured into these classes, which are then analyzed in their 'from-to' transition frequency. A forecast is created by choosing the class most transitioned into from the class recorded in the previous time period.

In order to test this idea, a time series of the monthly average temperature in San Juan with 22 records was used. In addition, the method proposed here was compared to the well-known naïve method. This method generates a forecast by choosing a value equal to the record in the previous time period. Using the sum of absolute differences as the performance metric (less is better), the proposed method obtained a value of 23.05 while the naïve method obtained a value of 25.00 based on the forecast of 21 temperature records.

The proposed method gave better results than the naïve method in our test, which is encouraging and prompts to further expand the development of this idea. The fractal scheme was used to identify

a small number of patterns in the time series under study that can be used to forecast in a very simple manner.

A fractal scheme to support forecasting is an idea that should be expanded in its study as the first results point for it to be competitive and convenient. In the future, more than just one previous record will be considered and the proposed method will be computer coded for convenience.

Problems in finite precision arithmetic

Fabián Eleazar Calderón Gutiérrez, Escuela Especializada en Matemáticas, Ciencias y Tecnología de San Juan.
Daniel Melo Pantoja, Escuela Especializada en Matemáticas, Ciencias y Tecnología de San Juan.

Finite precision arithmetic limits the different processes a computer can do. Since each number is represented in an approximate form an error occurs. This error generates new errors when other operations between numbers are performed. The results can be unacceptable according to the context in use but historically we have lived the consequences of underestimate the errors in numerical calculations such as what happened with the missile detector Patriot in 1991, that failed to track and intercept a missile causing the death of 28 Americans in Saudi Arabia. In this sense, we are aware that computers have become an essential tool to solve a vast number of problems from engineering, physics, chemistry, mathematics, and science in general through numerical methods and it is necessary to have in mind that even though computers can do complex operations in just a few seconds, they are not perfect. Therefore, it is an extremely important task to all the professional community to pay attention to the analysis of errors in numerical processes in a meaningful way. This is the reason because in this research we studied and explained different examples of these errors to show the interested community the impact of errors in numerical calculations. We did this by showing two errors and analyzing the miscalculation they caused, which was done through two programs implemented in C++. Mathematical equivalent expressions and approximations by Taylor Polynomial are included in the programs to get better approximations. Therefore, the data generated in this work are important for the understanding of the errors and the conclusions you can get from the errors, since it helps you visualize when the error occurs, how it occurs and why it occurs. All of this will be demonstrated through the different processes and data shown in the research.

Acknowledgements: Prof. José Calderón, Dr. Liana Gutiérrez, Dr. Paul Castillo, Escuela Especializada en Matemáticas, Ciencias y Tecnología.

Keywords: finite precision, misconception, arithmetic, C++, programs

Estimation of the transmission, recovery rate and basic reproductive number in horses for Equine Rhinitis A Virus mathematical model SIR and numerical solutions

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Daniel Melo Pantoja, Specialized School of Mathematics, Science and Technology of San Juan.

This research contributed to the rate of transmission, recuperation, and basic reproductive number of the virus Equine Rhinitis A (ERAV) by applying the SIR model. There was a lack of mathematical modeling predicting the behavior of the virus in the SIR population. The importance of a mathematical model is that it can predict how transmissive the virus is, the percentage of recovered horses, and how many horses can propagate from a single infected horse. It is proposed a mathematical SIR model that can represent at a populational level the effect of ERAV, and its behavior. More specifically the estimation of the parameter values, the rate of transmission, and recuperation obtaining the basic reproductive number. Similarly, numerical simulations were performed to illustrate the dynamics of the disease in the total SIR population, which is a contribution to the knowledge of the disease and its impact on the population, considering that there is no biomathematical research on this virus. The most important contribution made in the research for ERAV was the application of the SIR model. The real-life horse population may not follow the model since the parameter values may change due to a variety of reasons (e.g. immunization, isolation, etc.). The model has the capacity to adapt to a change in the parameter values by updating its representations to the actual state of the horses and not the previous outdated values that were represented. This capacity of the SIR model permits the researcher to maintain the usage of the SIR model without contradiction. It can still be modeled and interpreted mathematically by replacing the value of the parameter with real-life values and then solving the equations.

Acknowledgements: This research was assisted in part by the Professor Freddy Medina and by the Partnership with the Specialized School of Mathematics, Science and Technology of San Juan.

Keywords: SIR model, beta, gamma, R0, numerical analysis and ERAV

Optimization-based analysis of microarray experiments involving CAR T-cells

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Traditionally, the basic pillars in cancer treatment are given through chemotherapy, radiotherapy and in some cases surgery. But with the advancement of science and technology, new ideas have emerged that little by little allow us to visualize innovative treatments for the fight against cancer. Cellular therapies have recently raised great hopes in cancer patients and CAR T cell therapies have entered one of the mainstreams for cancer treatment. Chimeric antigen receptor (CAR) T cell therapy is a way to obtain immune cells called T cells to fight cancer, these cells are characterized by being modified through genetic engineering in a laboratory so they can find and destroy cancer

cells. This type of treatment can be very helpful in treating some types of cancer, even when other treatments no longer work. We wish to implement a methodology that has been worked on in our applied optimization research group (AOG), to address the task of gene selection and structuring through mathematical optimization methods in microarray experiments involving CAR T cells. This methodology is called: Multiple Criteria Optimization (MCO), the purpose is to characterize the changes in gene expression induced by the presence of a CAR in cells, select a set of genes that present the most significant changes in expression, perform an individual analysis of data sets under two performance measures, for our case, the mean, and the median. Finally, it is expected to have a series of potential biomarkers associated with the treatment with CAR T cells. We emphasize that the application of these methods has the possibility of analyzing multiple experiments simultaneously with deterministic repeatability and user-independent objectivity.

Acknowledgements: This work was done in collaboration with A. A. Sanchez-Castro (University of Puerto Rico at Mayagüez), A. C. Rentas-Echeverria (University of Puerto Rico at Mayagüez), M. Cabrera-Ríos (University of Puerto Rico at Mayagüez), and C. E. Isaza (University of Puerto Rico at Mayagüez)..

Keywords: CAR T, biomarker, MCO

Actions and factorizations

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In 2011, Anderson and Frazier defined the notion of generalized factorizations using a restriction of the multiplicative operation with respect to a symmetric relation τ on the non-zero and non-unit elements of an integral domain. This notion can be interpreted as allowing to multiply only the elements that are related with respect to τ. They defined three relations not seen before. However, the nature of the properties of relations need further study. In this research project we defined an action on a set of relations, coordinate-wise. We expect to expand our toolset to characterize properties of relations and the potential implications they might have in the theory of generalized factorizations.

Acknowledgements: This research was supported by the Puerto Rico Louis Stokes Alliance for Minority Participation (PR-LSAMP) Program.

Keywords: relations, action, factorizations

An iterative multicriteria simulation optimization method applied to true experiments

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The iterative multicriteria simulation optimization (iMCSO) method was developed originally to converge to system configurations that have the best balances between multiple performance measures based upon its simulation representation. It achieves this goal by the sequential construction of the Pareto efficient frontier aided by experimental design and second order model approximations on each iteration. In this talk we discuss the use of iMCSO to approach true experimental sequences as opposed to simulated ones. To this end, the configuration of a plastic strap characterized by its width and length is sought through the simultaneous minimization of its linear deformation and strap weight, two metrics in conflict. The experimental setting involves building straps following the dimensions prescribed by iMCSO and uses them to hang a mass of five pounds for 30 seconds, after which their length changes and their weights are measured. The method is applied using a code of recent creation that automates its iterative process. This work will demonstrate the performance of the method to approach true experiments.

Acknowledgements: This work was done in collaboration with M. E. Maldonado Toledo University of Puerto Rico at Mayagüez), R. Vázquez Rosario (University of Puerto Rico at Mayagüez), and M. Cabrera-Ríos (University of Puerto Rico at Mayagüez).

Keywords: experimental design, metamodeling, multicriteria optimization, Pareto efficient frontier

Estimación de parámetros, número reproductivo básico y análisis numérico del modelo SIR aplicado a la enfermedad roséola infantil

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La roséola infantil es una enfermedad que está clasificado como un exantema súbito. Es decir, esta enfermedad o virus suele afectar a infantes entre las edades de 4 meses y 2 años. Su proceso de incubación consta de 5 a 15 días, con un promedio de 12 días. Asimismo, el 95los 2 años, especialmente entre los 6 y 8 meses de edad. Dicho virus usualmente produce síntomas de fiebre repentinos, con duración de 3 a 5 días. Se pueden presentar otros tipos de síntomas, incluso pueden aparecer marcas de enrojecimiento en la piel en los pacientes, diarrea, párpados edematosos, agrandamiento glandular e irritabilidad. Aun así, todavía no he han encontrado métodos para prevenir la roséola infantil. Incluso, tampoco se ha planteado un análisis del mismo virus mediante la biomatemática.

Se realizo un modelo matemático identificando los casos susceptibles (S), infectados (I), recuperados (R), para estudiar la dinámica de la infección a nivel poblacional, en el cual se realizó una estimación de parámetros encontrando la tasa de transmisión, recuperación y el número reproductivo básico, así como también sus soluciones numéricas. En el análisis numérico se presenta los comportamientos numéricos y las gráficas que ilustran la dinámica entre los susceptibles, infectados y recuperados de la enfermedad roséola infantil y el impacto del virus en la población.

Agradecimientos: Especialmente a la Escuela especializada en Matemáticas Ciencias y Tecnología del municipio de San Juan, Puerto Rico.

Keywords: roséola infantil, modelo SIR, tasa de transmisión, tasa de recuperación y número básico reproductivo, análisis numérico

Constructing 2D watermarks by composition

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Alcibiades Bustillo, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

This study presented a method of constructing 2D periodic arrays by composing a 1D periodic array with a sequence of shifts calculated as a polynomial of order $n > 1$, $(\phi_1(x) \text{mod } p)$, with coefficients from $\mathbb{Z}_p$. The array construction is algebraic, based on finite fields, resulting in arrays with good correlation properties. This research illustrated such method using a Legendre sequence as a base array. The result- ing 2D array had a peak auto-correlation value of $p(p-1)$ and a non-peak auto-correlation value of $-p$, 0. Finally, the researchers showed the resistance of these watermarks to 180 degree rotation, Salt, and Gaussian attacks.

Statistical models for calcium signaling in Arabidopsis Plant

Daniel Rocha Clavijo, et al., University of Puerto Rico at Mayagüez.

Plants have receptors on their cell surface capable to recognize pathogens, to then trigger an immune response, and activate a local or systemic defense. Calcium (Ca2+) is one of the primary messengers in this response. The Ca2+ signature plays a crucial role in encoding and decoding the Ca2+ signal, enabling changes in cytosolic free Ca2+ to maintain nanomolar amounts. This work is part of the NSF EMBRIO institute, and it aims to analyze data of Arabidopsis epidermal cell cytosolic calcium levels over time, relating to three layers of cells: initiator cells (0), standby cells (1), and sub-standby cells (2). The experiment is aimed to determine the dynamics of cytosolic calcium in the plant immune response in different layers of cells when the bacterial peptide Flagellin 22 (flg22) is applied. The methodology consists of, i) organizing the elements of the experiment as a system representation (inputs, processes, outputs), ii) generating the data and fitting statistical models and iii) evaluating models using the Akaike information criterion (AIC) and the Bayesian information criterion (BIC) to select a parsimonious representation. Finally, the ANOVA of smooth functions allows determining the behavior of the average curves of the calcium signal through time

for the different layers of cells. An R shiny interface was also developed to support this work as demonstrated here.

Acknowledgements: This research was done in collaboration with L. Teran Herrera (University of Puerto Rico at Mayagüez), J. R. Helwig (Purdue University, West Lafayette), W. Zhang (Purdue University, West Lafayette), C. J. Staiger (Purdue University, West Lafayette), C. Isaza (University of Puerto Rico at Mayagüez), and M. Cabrera (University of Puerto Rico at Mayagüez). This work is based upon efforts supported by the EMBRIO Institute, contract #2120200, a National Science Foundation (NSF) Biology Integration Institute.

Keywords: Calcium signature, epidermal cell, peptide Flagellin 22, Akaike information criterion, Bayesian information criterion, ANOVA of smooth functions

Analyzing mitigation strategies in Puerto Rico for the COVID-19 pandemic utilizing an epidemiological mathematical model

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Maytee Cruz Aponte, Department of Mathematics-Physics, University of Puerto Rico at Cayey.

Throughout the covid 19 pandemic, Puerto Rico's government has established a series of preventative measures to avoid the spread of the virus throughout the island. These measures have had a series of effects in the contagion rates that have not yet been studied. This study aims to replicate and analyze these conditions in a metapopulation study that allow us to visualize which government measures were efficient in reducing contagion and how to apply them in future pandemics. To achieve this, a modified Susceptible, Infected, Recovered (SIR) model was used to study the evolution of the SARS-Covid-2 pandemic. The added categories were: Exposed, Asymptomatic, Hospitalized, Convalescent. These modifications more accurately represent difficulties established by the COVID-19 pandemic. To acquire the information necessary to observe these variables, traffic data that describes the influx of people from one town to another in Guayama's police district, a southern region of Puerto Rico, was collected from public records of the Department of Transportation and Public Works. Based on the epidemiological model established, utilizing mitigation strategies alone only reduced the incidence of infections but had no effect on the disease's prevalence across the population. On the other hand, a pharmaceutical approach, such as the distribution of vaccines, was able to reduce the prevalence of the disease in the population. Therefore, this study alludes to show that mitigation strategies that allows the research, development and distribution of vaccines are the best approach to avoid the most negative effects of a pandemic.

Keywords: COVID-19, epidemiological model, mitigation, interventions, vaccines

Video monitoring of behavior assays of honeybees

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The economic impact of apiculture and the ecological significance of honeybees are among the reasons why the study of these pollinators has garnered interest in the scientific community. Historically, biologists have been limited to the manual analysis of animal movement data. Nowadays, it is possible to facilitate the study of animal behavior via Machine Learning and assigning a computer to the task. To this end, this research evaluates the use of DeepLabCut (DLC), a software package that employs TensorFlow-based neural networks, in bee pose estimation. A DLC neural network model was trained on frames captured from videos of bees. This model was then used to estimate the poses of the bee's head, thorax, and abdomen throughout a novel video, thus enabling the quantification of the effect of alcohol or other substances on honeybees' behavior in tubes in controlled lab conditions. The resulting pose data was refined using k-means clustering to further subdivide automatically the pose data into individual tracks corresponding to each tube, thus producing time-series of the behavior of each individual over the full length of the video.

Acknowledgements: This research was done in collaboration with J. Hernández Campos (University of Puerto Rico at Río Piedras), R. Mégret (University of Puerto Rico at Río Piedras). R. A. Espaillat Perez (University of Puerto Rico at Río Piedras), and J. L. Agosto-Rivera (University of Puerto Rico at Río Piedras). This work is supported by award #2021-67014-34999 from USDA-NIFA: "Deep-Pollinator: Enabling Large-Scale Video Analysis Of Pollinator Behavior With Deep Learning".

Keywords: machine learning, deep learning, animal monitoring, computational ethology

Blood gene expression comparison between autism and schizophrenia through biooptimatics

Deiver Suárez-Gómez, et al., The Applied Optimization Group, University of Puerto Rico at Mayagüez.

Neurodevelopmental disorders like schizophrenia (SCZ) and autism spectrum disorder (ASD) are characterized by causing social and cognitive impairments. Clinically, these conditions are distinct. But different studies have demonstrated that there are genetic, environmental, and symptomatic similarities between them. This work aims to better understand the genetic expression differences and overlaps between ASD and SCZ through a joint analysis of publicly available gene expression microarray datasets. For this goal, biooptimatics proposes a pipeline of three mathematical optimization methods: Multi-Criteria Optimization (MCO), Minimum Spanning Tree (MST), and Optimal Group Formation (OGF). These three methods provide: lists of genes with maximal expression changes, signaling pathways through a non-cyclic maximal correlation structure, and groups of genes and biological processes under global optimality conditions, respectively. We present analyzes of two datasets GSE18123(blood) in ASD and GSE38481(Whole blood) in SCZ, these analyzes were

performed by individual analysis of each dataset and condition and by meta-analysis dividing the datasets by sex. The results showed common genetic expression changes for ASD and SCZ, such as the RPS27 and VNN2 genes; common biological pathways affected in both conditions, such as regulation of the ribosome, malaria infection, and the actin cytoskeleton; and well-established sets of genes and biological processes, that define structures between conditions, genes, and biological processes.

Acknowledgements: This research was done in collaboration with S. R Mendez Cruz (University of Puerto Rico at Mayagüez) , K. Rivera-Rodriguez (University of Puerto Rico at Mayagüez), I. Narváez-Bandera (University of Puerto Rico at Mayagüez), M. Cabrera-Río (University of Puerto Rico at Mayagüez), and C. E. Isaza (University of Puerto Rico at Mayagüez).

Keywords: multi-criteria optimization, minimum spanning tree, optimal group formation, neurodevelopmental disorders, genetic expression differences and microarray datasets

Index